An INTRODUCTION to

SWITCHING ADAPTIVE CONTROL

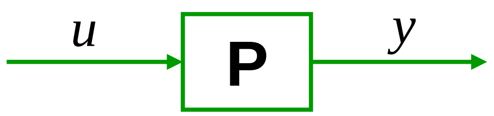
Daniel Liberzon

Coordinated Science Laboratory and
Dept. of Electrical & Computer Eng.,
Univ. of Illinois at Urbana-Champaign

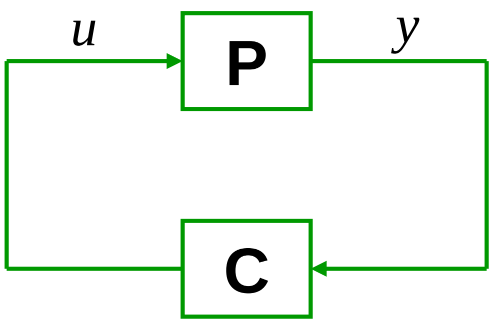
Based on joint work with J.P. Hespanha (UCSB) and A.S. Morse (Yale)

SWITCHING CONTROL

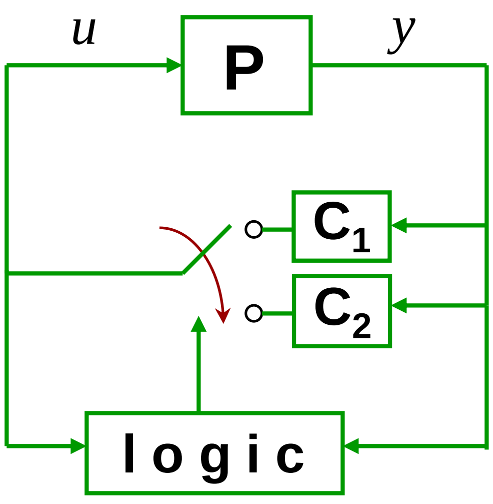
Plant:



Classical continuous feedback paradigm:



But logical decisions are often necessary:



REASONS for SWITCHING

- Nature of the control problem
- Sensor or actuator limitations
- Large modeling uncertainty
- Combinations of the above

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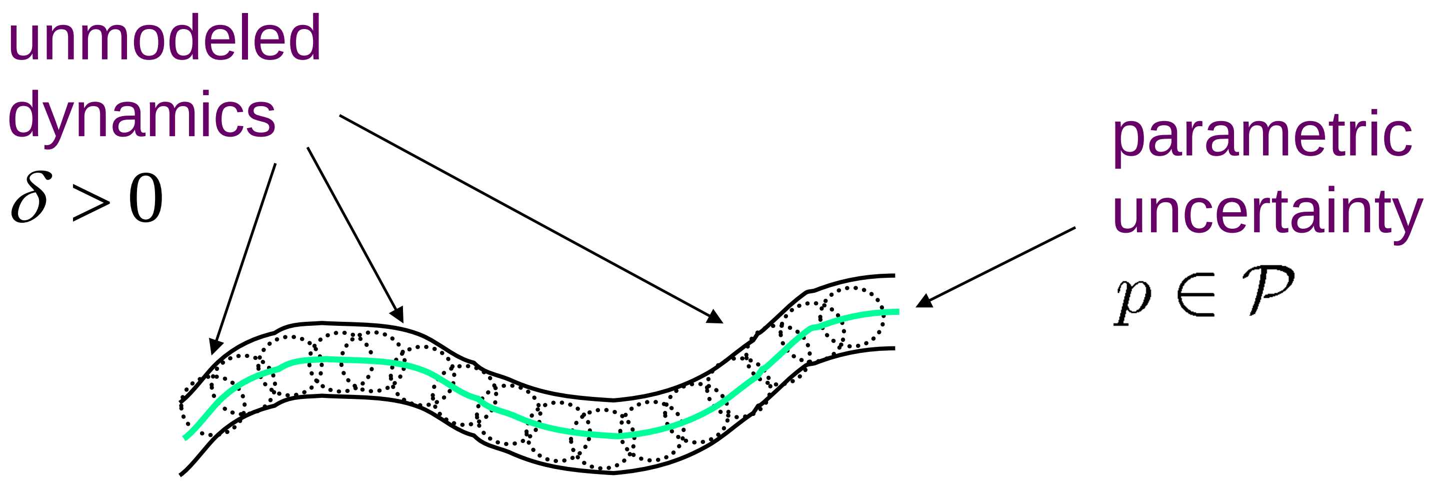
MODELING UNCERTAINTY



$$\mathcal{F} = \bigcup_{p \in \mathcal{P}} \mathcal{F}_p$$

Also, noise n and disturbance d

Adaptive control (continuous tuning)
vs. supervisory control (switching)

EXAMPLE

Scalar system:

$$\dot{y} = y^2 + p^* u$$

$$\mathcal{P} = [-10, -0.1] \cup [0.1, 10]$$

$p^* \in \mathcal{P}$, otherwise unknown

(purely parametric uncertainty)

$$u = -\frac{1}{p^*}(y^2 + y) \quad \Rightarrow \quad \dot{y} = -y \qquad \text{stable } \checkmark$$

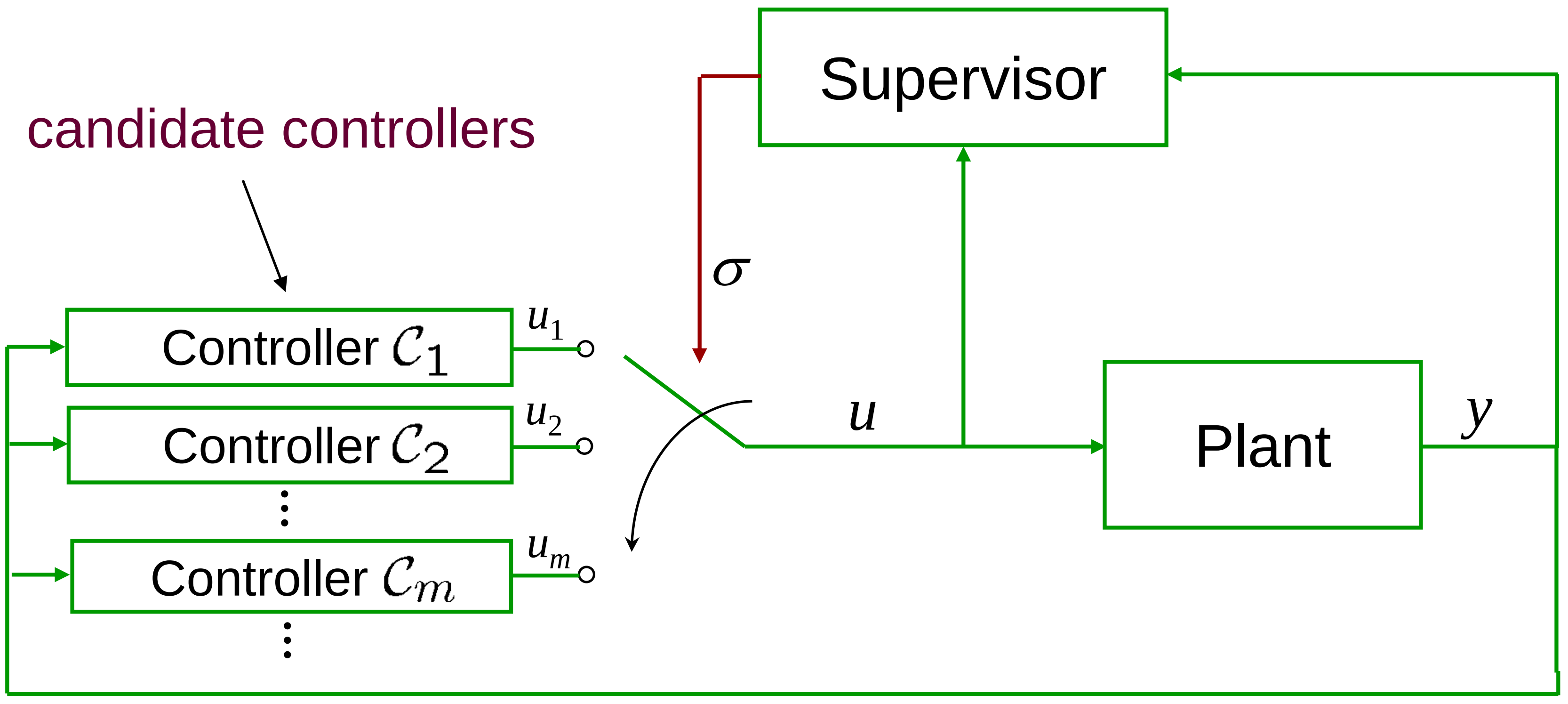
not implementable

Controller family: $u_q = -\frac{1}{q}(y^2 + y), \;\; q \in \mathcal{P}$

Could also take $u_q, \;\; q \in \mathcal{Q} = \{-1, 1\}$

controller index set

SUPERVISORY CONTROL ARCHITECTURE



σ – switching signal, takes values in $\mathcal{Q}$

$\mathcal{C}_\sigma$ – switching controller

TYPES of SUPERVISION

- Prescheduled (prerouted)
- Performance-based (direct)
- Estimator-based (indirect)

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OUTLINE

- Basic components of supervisor
- Design objectives and general analysis
- Achieving the design objectives (highlights)

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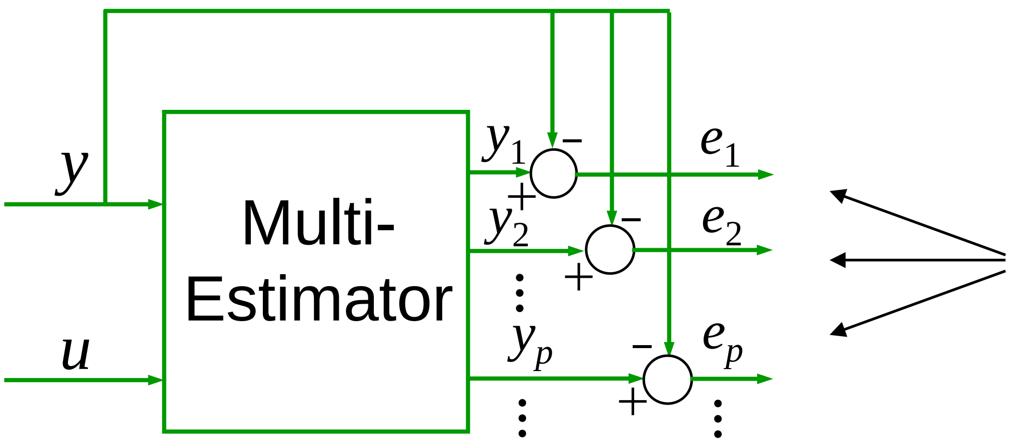
SUPERVISOR



estimation errors:

$$e_p := y_p - y, \;\; p \in \mathcal{P}$$

Want e_{p^*} to be small

Then e_p small indicates $p = p^*$ likely

EXAMPLE

$$\dot{y} = y^2 + p^* u$$

Multi-estimator:

$$\dot{y}_p = -(y_p - y) + y^2 + pu, \;\; p \in \mathcal{P}$$

$$e_p = y_p - y, \;\; p \in \mathcal{P}$$

$$\Downarrow$$

$\dot{e}_{p*} = -e_{p*}$ => $e_{p*} \longrightarrow 0$ exp fast $\forall\, u$

EXAMPLE

$$\dot{y} = y^2 + p^* u - d$$

disturbance

Multi-estimator:

$$\dot{y}_p = -(y_p - y) + y^2 + pu, \;\; p \in \mathcal{P}$$

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$$\Downarrow$$

$\dot{e}_{p^*} = -e_{p^*} + d$ => $e_{p^*} \to d$ exp fast $\forall\, u$

STATE SHARING

$$\dot{y}_p = -(y_p - y) + y^2 + pu, \ \ p \in \mathcal{P}$$

Bad! Not implementable if $\mathcal{P}$ is infinite

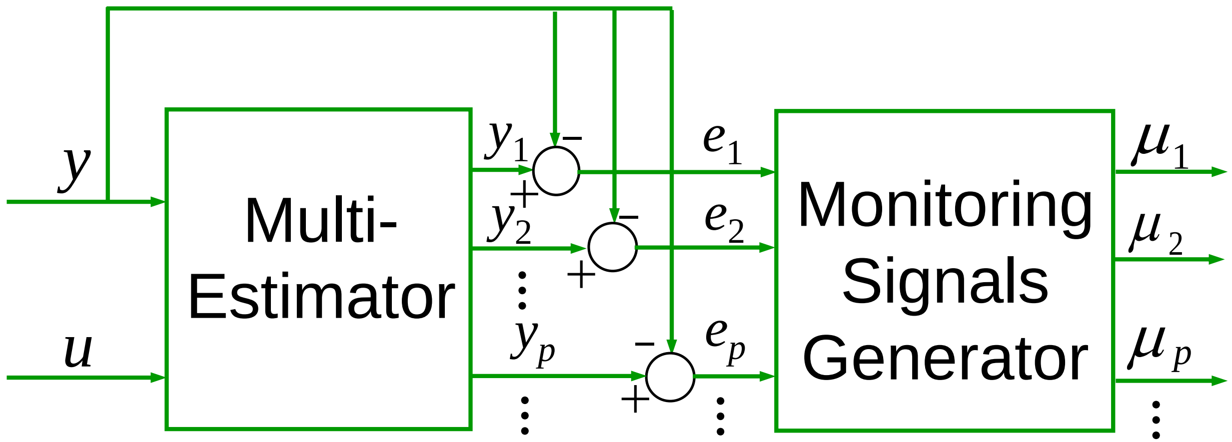
The system

$$\dot{z}_1 = -z_1 + y + y^2$$
$$\dot{z}_2 = -z_2 + u$$
$$y_p = z_1 + pz_2, \ \ p \in \mathcal{P}$$

produces the same signals

$$\dot{y}_p = \dot{z}_1 + p\dot{z}_2 = -z_1 + y + y^2 - pz_2 + pu \ = -y_p + y + y^2 + pu$$

SUPERVISOR



Examples:

$$\mu_p(t) = \int_0^t |e_p(\tau)|^2 d\tau \Leftrightarrow \dot{\mu}_p = |e_p|^2, \ \mu_p(0) = 0$$

$$\mu_p(t) = \int_0^t e^{-\lambda(t-\tau)} |e_p(\tau)|^2 d\tau \Leftrightarrow \dot{\mu}_p = -\lambda \mu_p + |e_p|^2, \ \mu_p(0) = 0$$

EXAMPLE

Multi-estimator:

$$\dot{z}_1 = -z_1 + y + y^2$$

$$\dot{z}_2 = -z_2 + u$$

$$y_p = z_1 + pz_2, \;\; p \in \mathcal{P}$$

$$\boxed{\dot{\mu}_p = e_p^2}$$ – can use state sharing

$$e_p^2 = (z_1 + pz_2 - y)^2 = (z_1 - y)^2 + 2pz_2(z_1 - y) + p^2 z_2^2$$

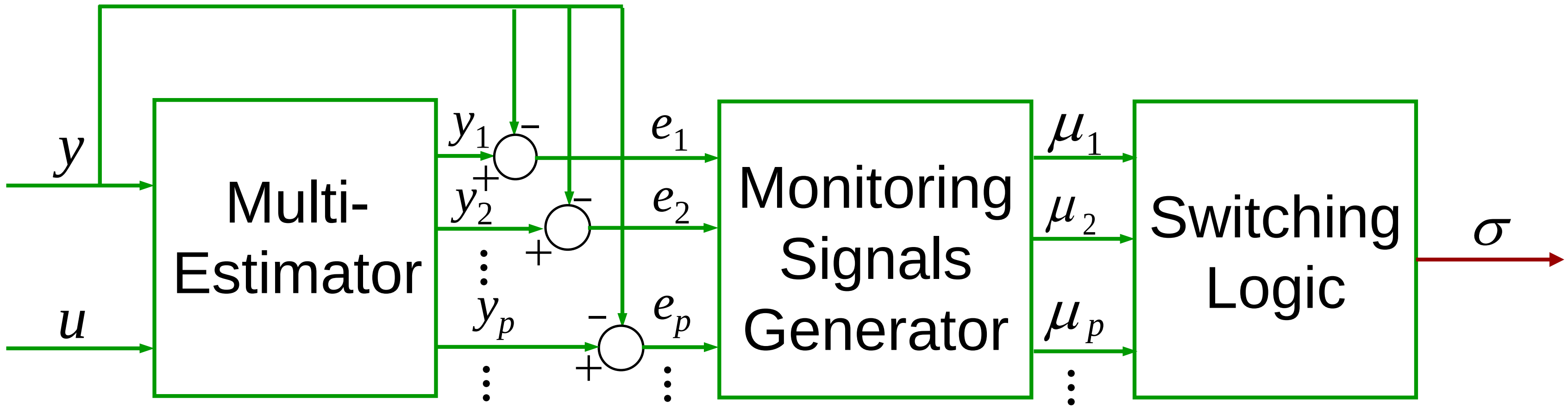
$$\dot{\eta}_1 = (z_1 - y)^2$$

$$\dot{\eta}_2 = 2z_2(z_1 - y)$$

$$\dot{\eta}_3 = z_2^2$$

$$\mu_p = \eta_1 + p\eta_2 + p^2\eta_3, \;\; p \in \mathcal{P}$$

SUPERVISOR



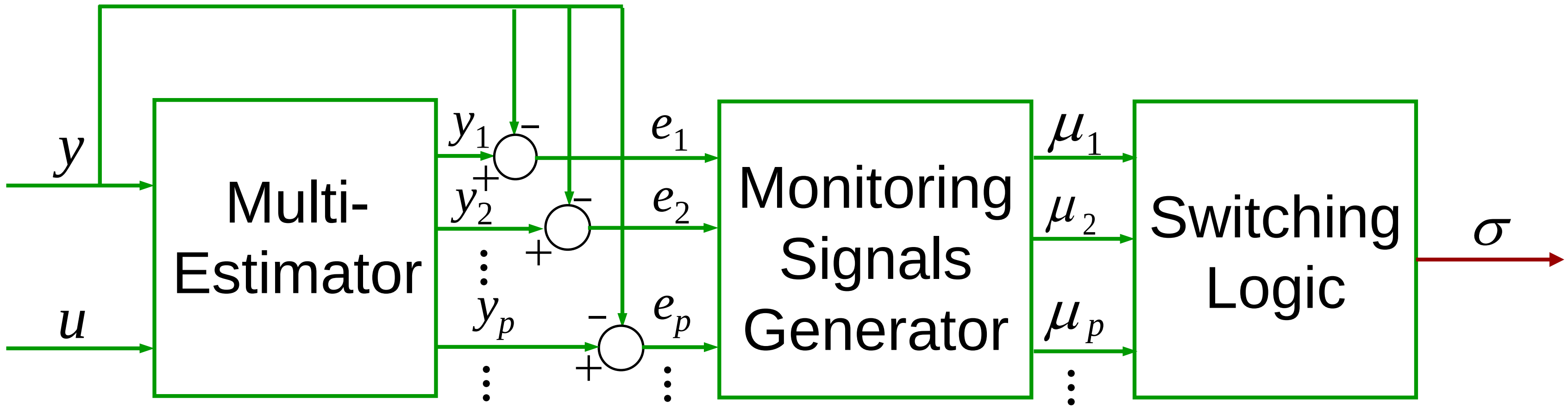
Basic idea: $\sigma(t) = \arg\min_{p \in \mathcal{P}} \mu_p(t)$

Justification? Plant $\in \mathcal{F} = \bigcup_{p \in \mathcal{P}} \mathcal{F}_p$, controllers: $\mathcal{C}_p,\ p \in \mathcal{P}$

μ_p small => e_p small => plant likely in $\mathcal{F}_p$ => $\mathcal{C}_p$ gives stable closed-loop system

("certainty equivalence")

SUPERVISOR



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μ_p small => e_p small ~~=>~~ plant likely in $\mathcal{F}_p$ => $\mathcal{C}_p$ gives stable closed-loop system

only know converse!

Need: e_p small => $\mathcal{C}_p$ gives stable closed-loop system

This is detectability w.r.t. e_p

DETECTABILITY

Linear case:

$$\dot{x} = A_q x \qquad \longleftarrow \text{ plant in closed loop with } \mathcal{C}_q$$

$$e_q = C_q x \qquad \longleftarrow \text{ view as output}$$

Want this system to be detectable

$$\Updownarrow$$

$$\boxed{e_q \to 0 \Rightarrow x \to 0}$$

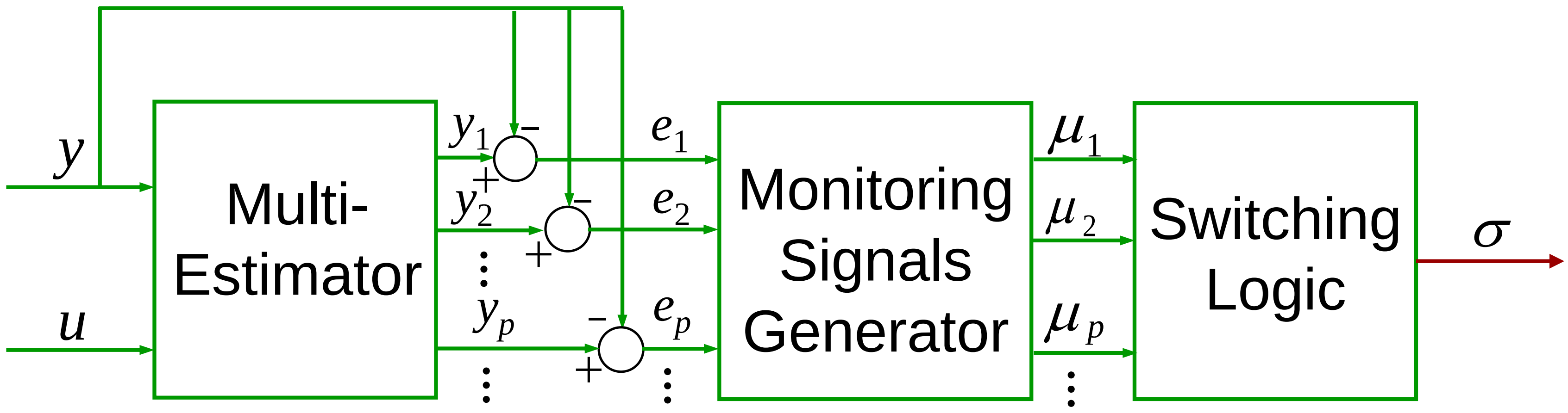
$$\Updownarrow$$

"output injection" matrix

$$\exists L_q : A_q - L_q C_q \text{ is Hurwitz}$$

$$\underbrace{\dot{x} = (A_q - L_q C_q) x}_{\text{asympt. stable}} + L_q e_q$$

SUPERVISOR



We know: e_{p^*} is small

Switching logic (roughly): $\sigma(t) = \arg\min_{p \in \mathcal{P}} \mu_p(t)$

This (hopefully) guarantees that e_σ is small

Need: e_σ small => stable closed-loop switched system

This is switched detectability

DETECTABILITY under SWITCHING

Switched system: $\dot{x} = A_\sigma x$ $\longleftarrow$ plant in closed loop with $\mathcal{C}_\sigma$

$e_\sigma = C_\sigma x$ $\longleftarrow$ view as output

Want this system to be detectable: $e_\sigma \to 0 \Rightarrow x \to 0$

Assumed detectable for each frozen value of σ

Output injection:

$$\underbrace{\dot{x} = (A_\sigma - L_\sigma C_\sigma)x}_{\text{need this to be asympt. stable}} + L_\sigma e_\sigma$$

Thus σ needs to be "non-destabilizing":

- switching stops in finite time
- slow switching (on the average)

SUMMARY of BASIC PROPERTIES

Multi-estimator:

1. At least one estimation error (e_{p^*}) is small

- $e_{p^*} \to 0 \ \forall u$ when $n = 0,\ d = 0,\ \delta = 0$
- e_{p^*} is bounded for bounded n & d

Candidate controllers:

2. For each $\mathcal{C}_q$, closed-loop system is detectable w.r.t. e_q

Switching logic:

3. e_σ is bounded in terms of the smallest e_p

4. Switched closed-loop system is detectable w.r.t. e_σ provided this is true for every frozen value of σ

conflicting: for 3, want to switch to $\arg\min_p \mu_p(t)$
for 4, want to switch slowly or stop

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Analysis:

1 + 3 => e_σ is small
2 + 4 => detectability w.r.t. e_σ

} => state is small ✓

OUTLINE

- Basic components of supervisor
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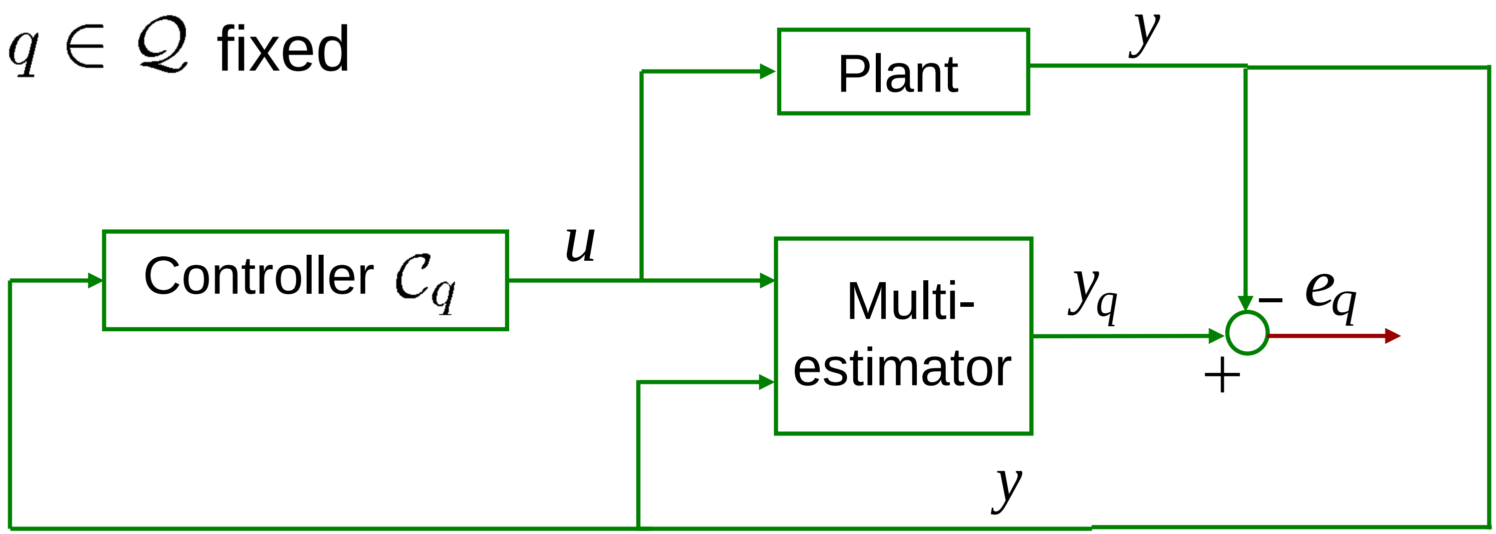
- Achieving the design objectives (highlights)

CANDIDATE CONTROLLERS

$q \in \mathcal{Q}$ fixed



CANDIDATE CONTROLLERS

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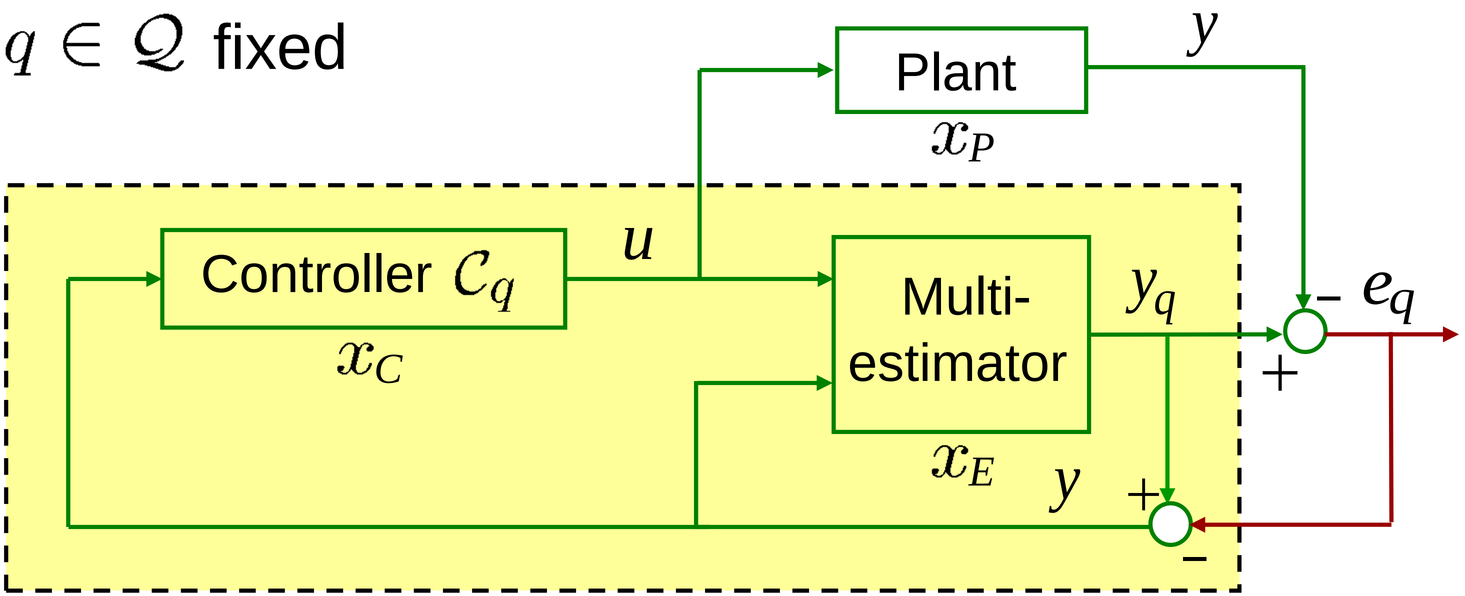


Linear: overall system is detectable w.r.t. e_q if

i. system inside the box is stable

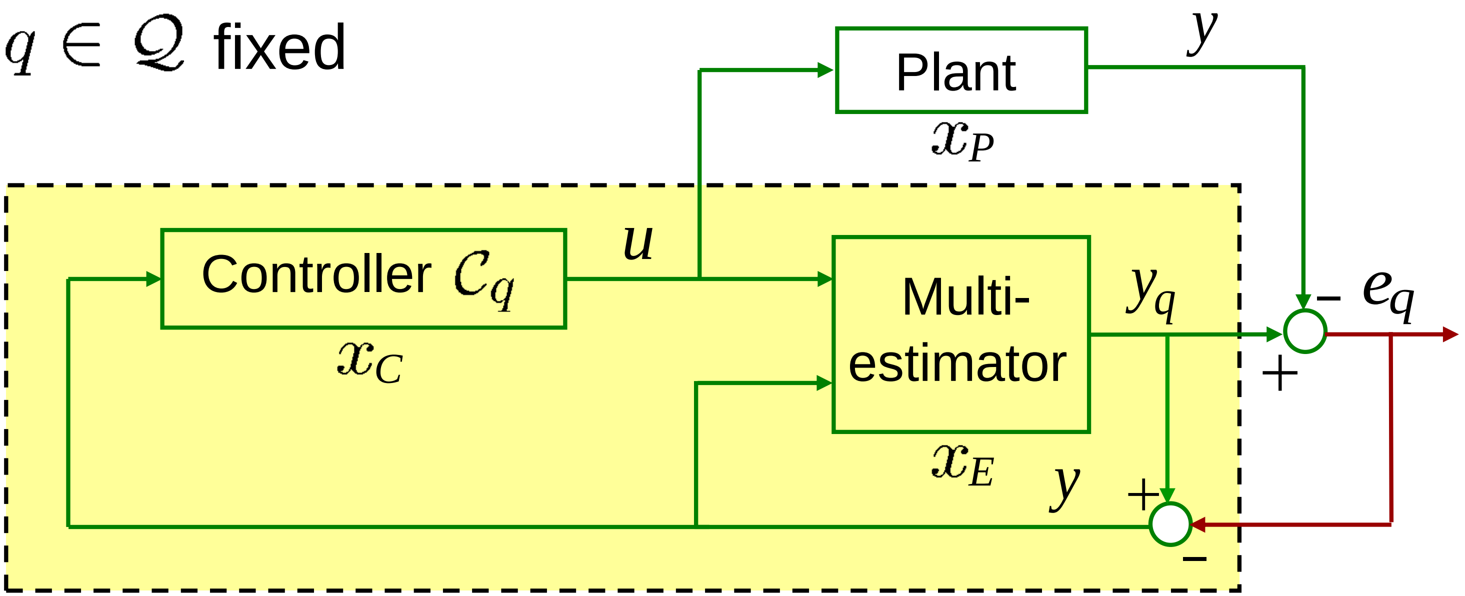
ii. plant is detectable

Need to show: $e_q \to 0$ => x_P, x_C, $x_E \to 0$

$e_q \to 0 \underset{\text{i}}{=>} x_C, x_E \to 0 => u, y_q \to 0 => y = y_q - e_q \to 0 \underset{\text{ii}}{=>} x_P \to 0$

CANDIDATE CONTROLLERS

$q \in \mathcal{Q}$ fixed



Linear: overall system is detectable w.r.t. e_q if

i. system inside the box is stable

ii. plant is detectable

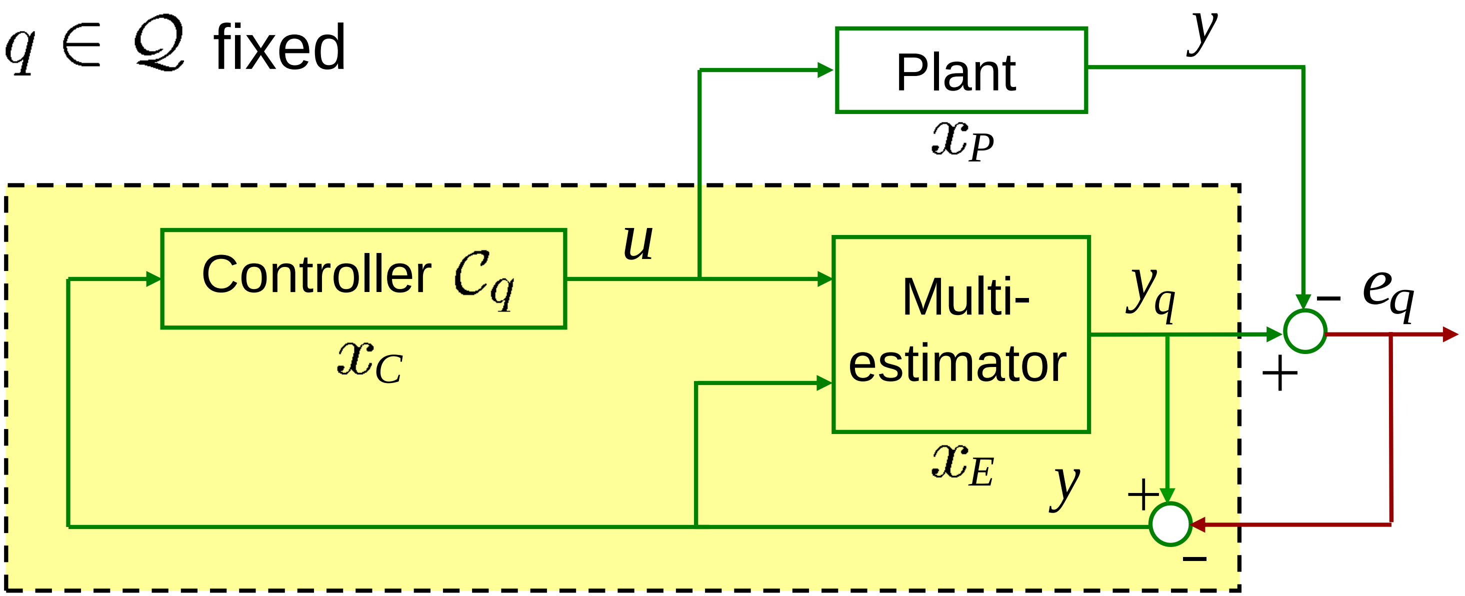
Nonlinear: same result holds if stability and detectability are interpreted in the ISS/OSS sense:

external signal

$$|x(t)| \leq \beta(|x(0)|, t) + \gamma\left(\|v\|_{[0,t]}\right)$$

CANDIDATE CONTROLLERS

$q \in \mathcal{Q}$ fixed



Linear: overall system is detectable w.r.t. e_q if

i. system inside the box is stable

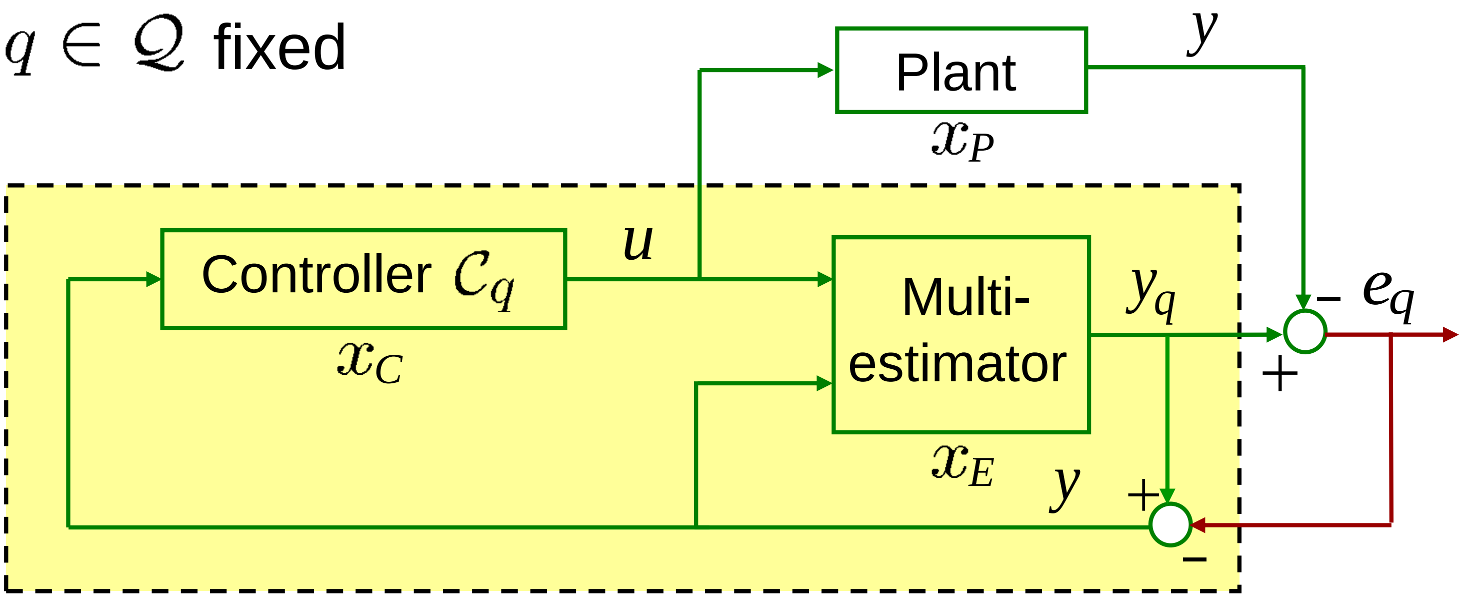
ii. plant is detectable

Nonlinear: same result holds if stability and detectability are interpreted in the integral-ISS/OSS sense:

$$|x(t)| \le \beta(|x(0)|, t) + \int_0^t \gamma(|v(\tau)|)d\tau$$

CANDIDATE CONTROLLERS

$q \in \mathcal{Q}$ fixed



Linear: overall system is detectable w.r.t. e_q if

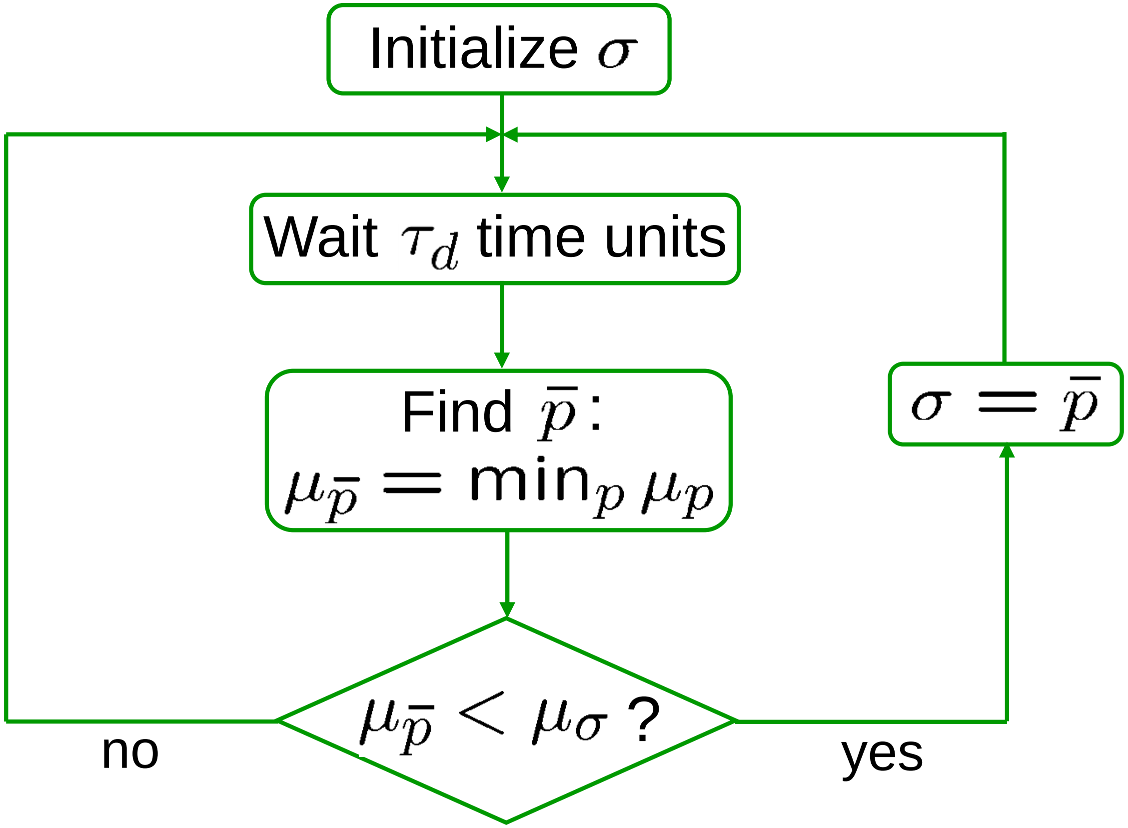
i. system inside the box is stable

ii. plant is detectable

For minimum-phase plants, it is enough to ask that the system inside the box be output-stabilized

SWITCHING LOGIC: DWELL-TIME

$\mu_p, \ \ p \in \mathcal{P}$ – monitoring signals $\qquad \tau_d > 0$ – dwell time



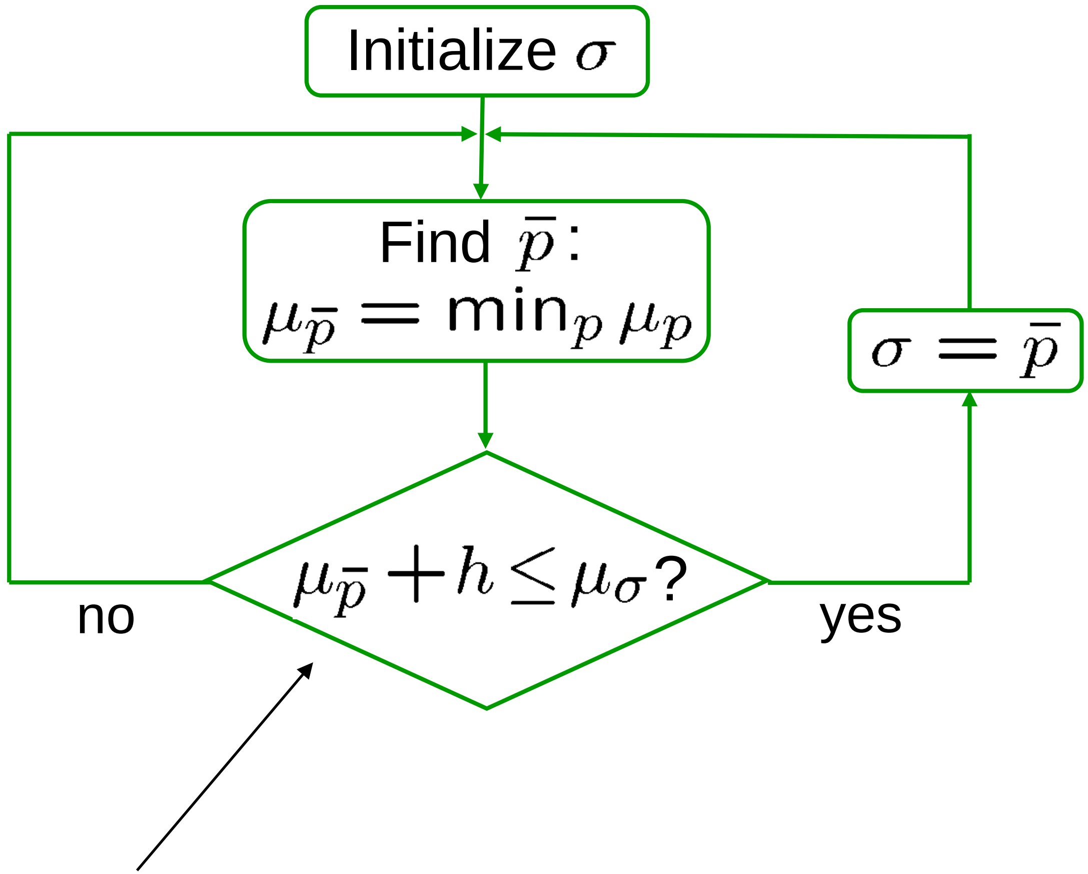
Detectability is preserved if τ_d is large enough ✓

Obtaining a bound on e_σ in terms of e_{p^*} is harder

Not suitable for nonlinear systems (finite escape)

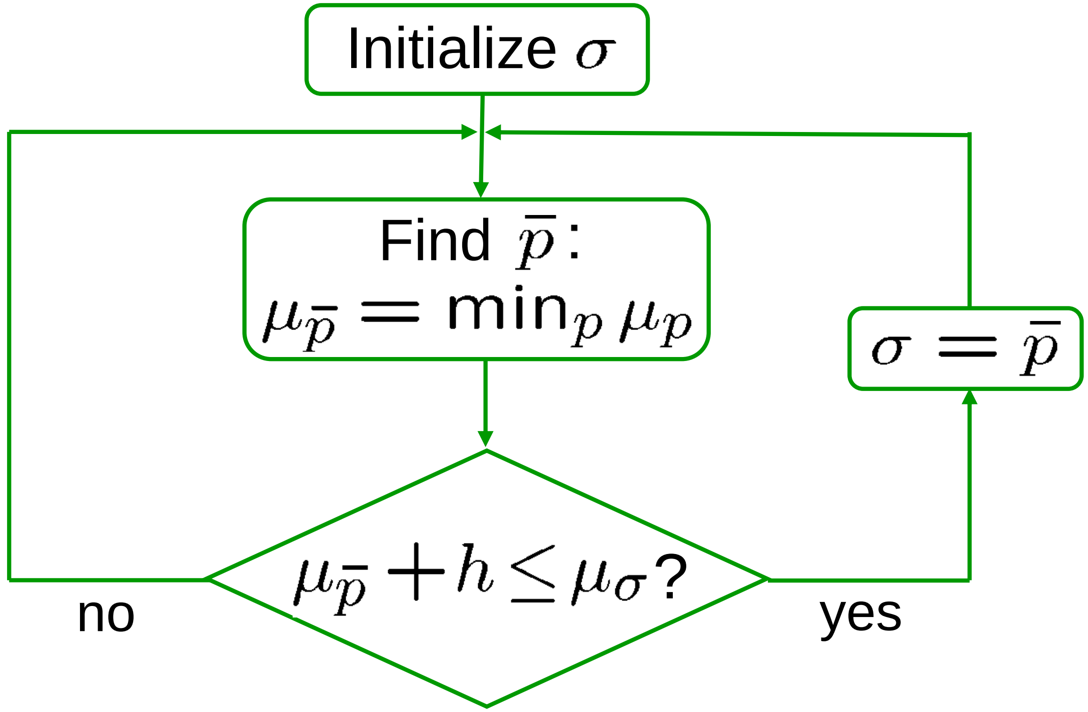
SWITCHING LOGIC: HYSTERESIS

$\mu_p, \; p \in \mathcal{P}$ – monitoring signals $h > 0$ – hysteresis constant



or $(1 + h)\mu_{\bar{p}} \leq \mu_\sigma$
(scale-independent)

SWITCHING LOGIC: HYSTERESIS



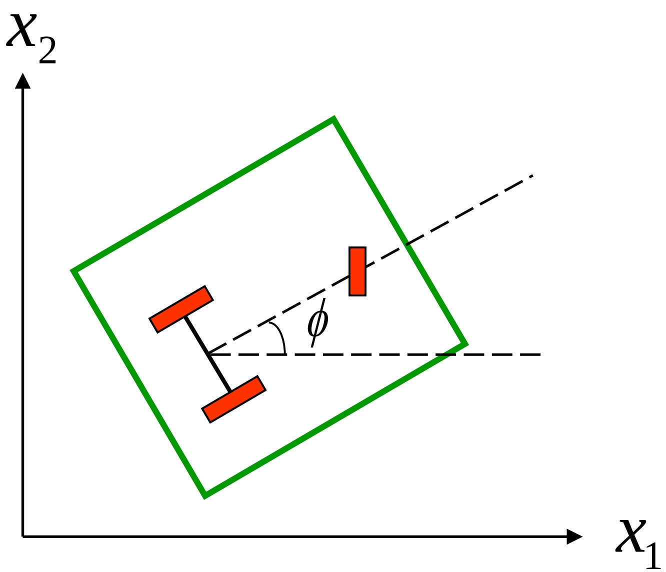
$\mathcal{P}$ finite, $\mu_p \uparrow$, μ_{p^*} bounded => switching stops in finite time

This applies to $\delta, n, d = 0$, $e_{p^*} \to 0$ exp fast, $\mu_p = \int |e_p|^2$

Linear, $\delta = 0$, n, d bounded => average dwell time $\tau_a(h)$

$$\int |e_\sigma|^2 \leq |\mathcal{P}|(1+h) \int |e_p|^2$$

TOY EXAMPLE: PARKING PROBLEM



$$\dot{x}_1 = p_1 w_1 \cos\phi$$

$$\dot{x}_2 = p_1 w_1 \sin\phi$$

$$\dot{\phi} = p_2 w_2$$

Unknown parameters p_1, p_2 correspond to the radius of rear wheels and distance between them