PROBABILISTIC SIGNAL PROCESSING ON GRAPHS

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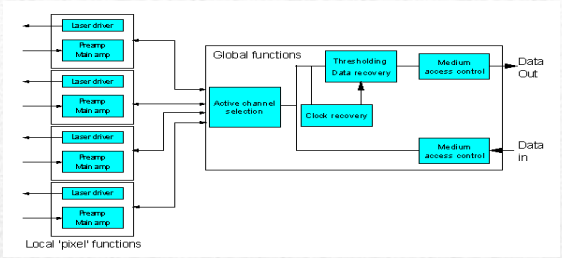
Outline:

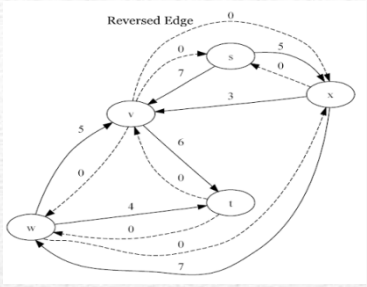
- Why graphs
- Manage uncertainties
- Types of graphs
- Examples: single block, small network, continuous densities «loopy graph»
- Learning in a graph – ML learning (EM)
- Application to learning non linear functions
- Application to Camera tracking
- Application to Deep multi-layer network
- The inference on the graph as a probabilistic computing machine
- Open Issues and future developments

Why Graphs ?:

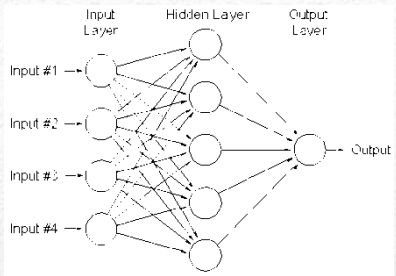
We think on graphs!



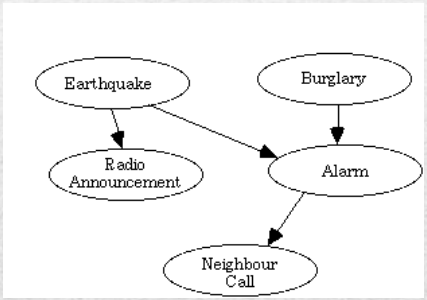
Signal flow diagram



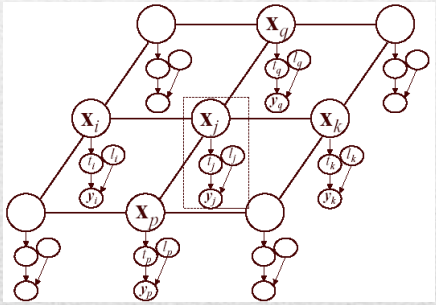
State transition graph



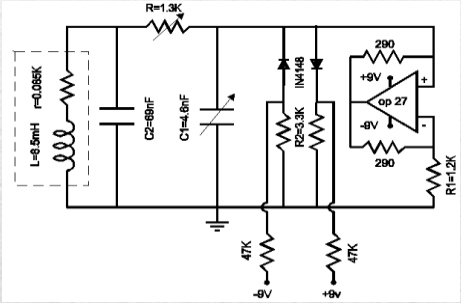
Neural network



Bayesian reasoning



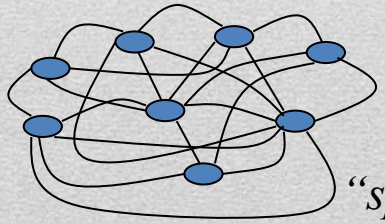
Markov random field



Circuit diagram

The graph represents most of our a priori knowledge about a problem.

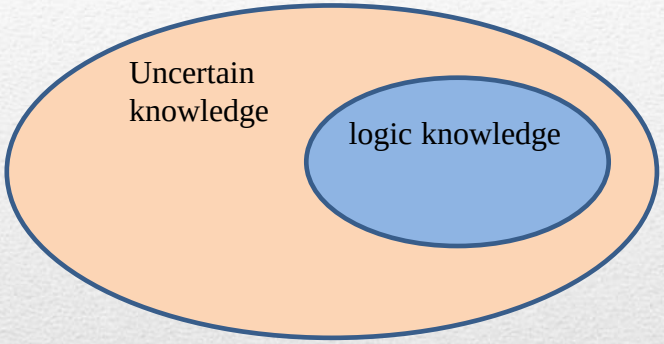
If everything were connected to everything:



"spaghetti"

Intelligence = manage uncertainties:

Smart fusion consists in providing the best answer with any available information, with both discrete and continuous variables, noise, erasures, errors, hard logic, weak syllogisms, etc.



….The "new" perception amounts to the recognition that the mathematical rules of probability theory are not merely rules for calculating frequencies of "random variables"; they are also the unique consistent rules for conducting inference ***(i.e. plausible reasoning) of any kind****…*

…..each of his (Kolmogorov's) axioms turns out to be, for all practical purposes, derivable from the Polya-Cox desiderata of rationality and consistency. In short, ***we regard our system of probability as not contradicting Kolmogorov's; but rather seeking a deeper logical foundation*** *that permits its extension in the directions that are needed for modern applications….*

Jaynes E.T., Probability Theory: The Logic of Science, Cambridge University Press (2003)

Model dependencies:

Given N (deterministic or random) variables $X_1, X_2, ..., X_N$, model all possible dependencies

$$p(X_1 X_2 ... X_N) \quad \text{(joint pdf)}$$

Knowledge of the structural dependencies is in the factorization (graph) of p

Group the variables in M subsets $\{\mathcal{S}_c, c =, 1, ..., M\}$ (cliques)

$$p(X_1 X_2 ... X_N) \propto \prod_{c=1}^{M} \Psi_c \left(\bigcap_{j \in \mathcal{S}_c} X_j \right) \quad (\Psi_c \text{ potential functions})$$

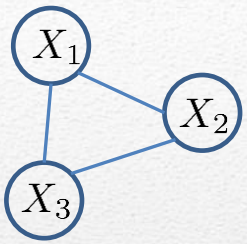
$$p(X_1 X_2 ... X_N) \propto exp \left(\sum_{c=1}^{M} \phi_c \left(\bigcap_{j \in \mathcal{S}_c} X_j \right) \right) \quad (\phi_c \text{ energy functions})$$

Use the chain rule

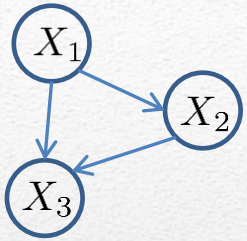
$$p(X_1 X_2 ... X_N) = \prod_{j=1}^{N} p(X_j | X_{j+1} ... X_N)$$

drop some conditioning variables using conditional independence assumptions. There are $N!$ ways of rearraging the variables.

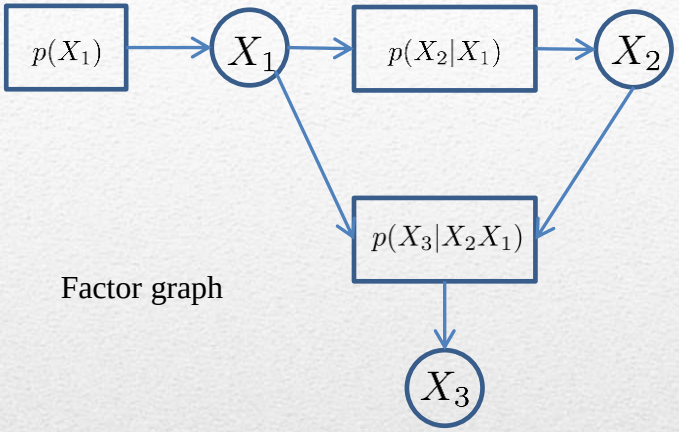
What kind of graph:



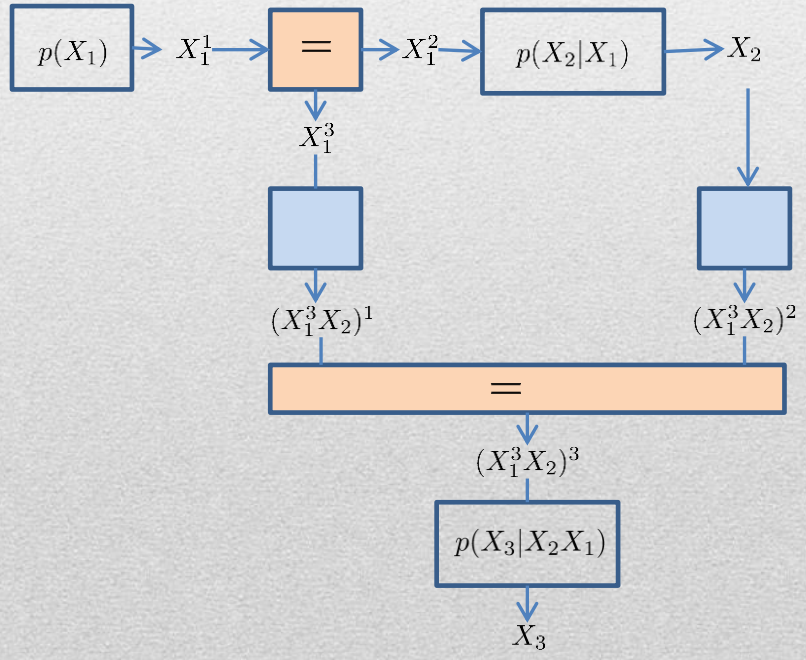
Undirected graph



Directed graph



Factor graph



Normal Graph
(Forney's style)

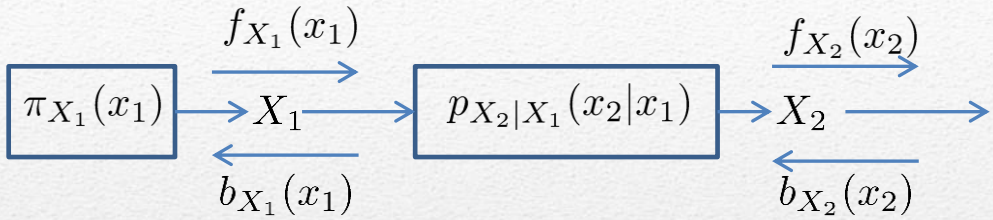
More workable model:

- Much easier message propagation
- Unique rules for learning

(this example has a loop)

Example 1: (to see how message propagation works)

$$p_{X_1X_2}(x_1x_2) = p_{X_2|X_1}(x_2|x_1)\pi_{X_1}(x_1)$$



Possible use: observe $X_2 = x_2^0$ and infer on X_1

$$p(x_1|X_2 = x_2^0) = \frac{p(X_2 = x_2^0|x_1)\pi_{X_1}(x_1)}{p(X_2 = x_2^0)} \propto \underbrace{p(X_2 = x_2^0|x_1)}_{b_{X_1}(x_1)} \underbrace{\pi_{X_1}(x_1)}_{f_{X_1}(x_1)}$$

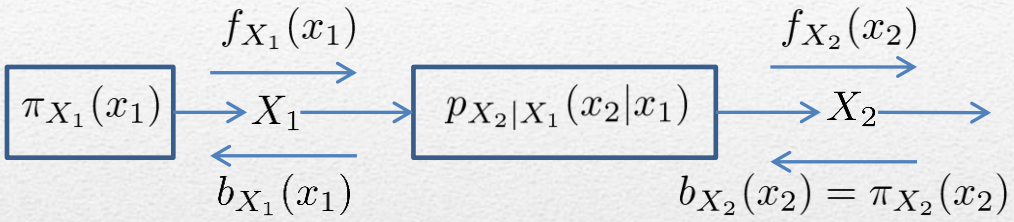
$$b_{X_1}(x_1) = \int_{\mathcal{X}_2} p_{X_2|X_1}(x_2|x_1) \underbrace{\delta(x_2 - x_2^0)}_{b_{X_2}(x_2)} dx_2$$

$$f_{X_2}(x_2) = \int_{\mathcal{X}_1} p_{X_2|X_1}(x_2|x_1) \underbrace{\pi_{X_1}(x_1)}_{f_{X_1}(x_1)} dx_1$$

Example 1: (cont.)

$$p_{X_1X_2}(x_1x_2) = p_{X_2|X_1}(x_2|x_1)\pi_{X_1}(x_1)$$

Possible use: use soft knowledge on X_2, $\pi_{X_2}(x_2)$ and infer on X_1

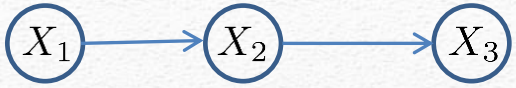


$$p(x_1|\pi_{X_2}) \stackrel{def}{=} \int_{\mathcal{X}_2} p(x_1|X_2 = x_2)\pi_{X_2}(x_2)dx_2 \propto \underbrace{\pi_{X_1}(x_1)}_{f_{X_1}(x_1)} \underbrace{\int_{\mathcal{X}_2} p_{X_2|X_1}(x_2|x_1)\underbrace{\pi_{X_2}(x_2)}_{b_{X_2}(x_2)}dx_2}_{b_{X_1}(x_1)}$$

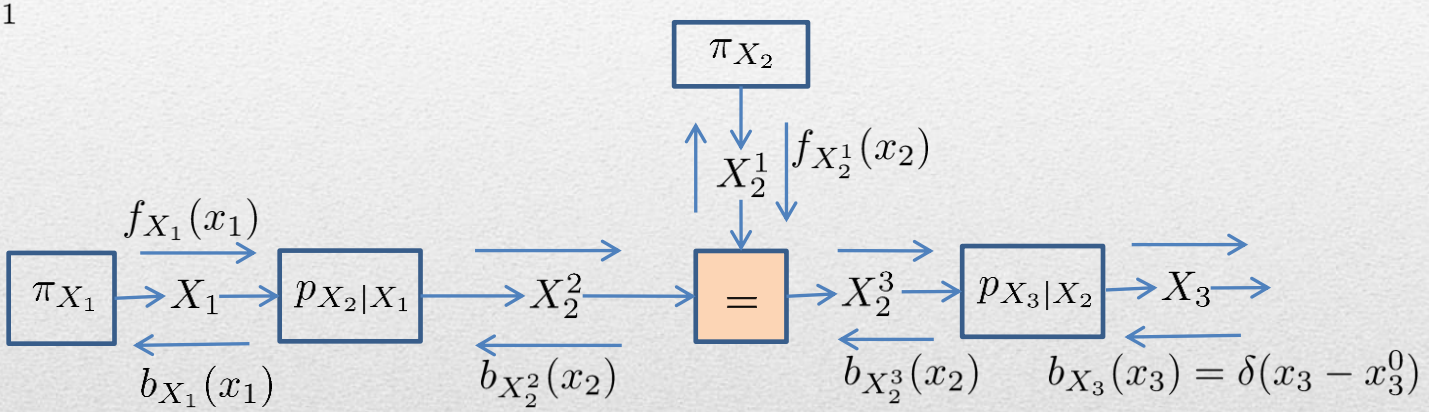
Sum-Product rule

Example 2:

$$p_{X_1X_2X_3}(x_1x_2x_3) = p_{X_3|X_2}(x_3|x_2)p_{X_2|X_1}(x_2|x_1)\pi_{X_1}(x_1)$$



Possible use: observe $X_3 = x_3^0$, use soft knowledge on X_2, $\pi_{X_2}(x_2)$ and infer on X_1

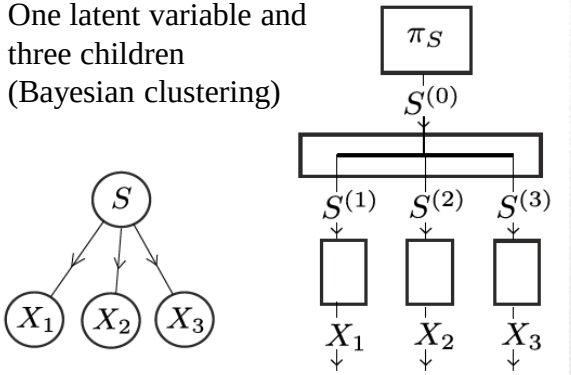


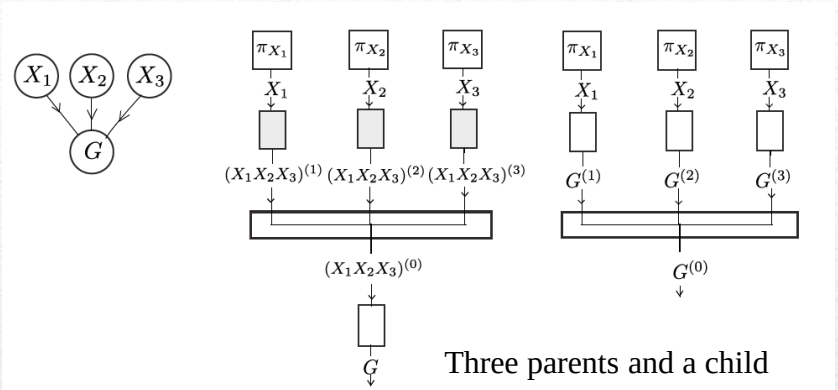
Insert a T-junction in the probability pipeline

$$f_{X_1}(x_1) = \pi_{X_1}(x_1);\ f_{X_2^1}(x_2) = \pi_{X_2}(x_2);$$
$$b_{X_2^3}(x_2) \propto \int_{\mathcal{X}_3} p_{X_3|X_2}(x_3|x_2)b_{X_3}(x_3)dx_3;\ \text{(sum)}$$
$$b_{X_2^2}(x_2) \propto f_{X_2^1}(x_2)b_{X_2^3}(x_2);\ \text{(product)}$$
$$b_{X_1}(x_1) \propto \int_{\mathcal{X}_2} p_{X_2|X_1}(x_2|x_1)b_{X_2^2}(x_2)dx_2;\ \text{(sum)}$$
$$p(x_1|\pi_{X_2}, X_3 = x_3^0) \propto f_{X_1}(x_1)b_{X_1}(x_1)\ \text{(product)}$$

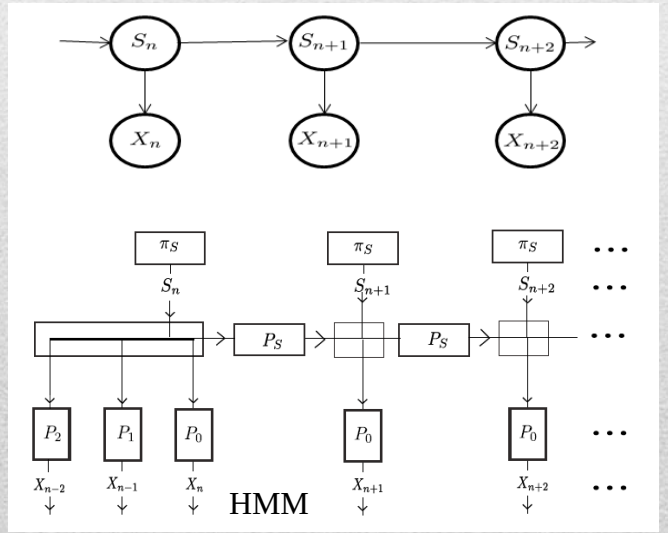
More examples:

One latent variable and three children (Bayesian clustering)

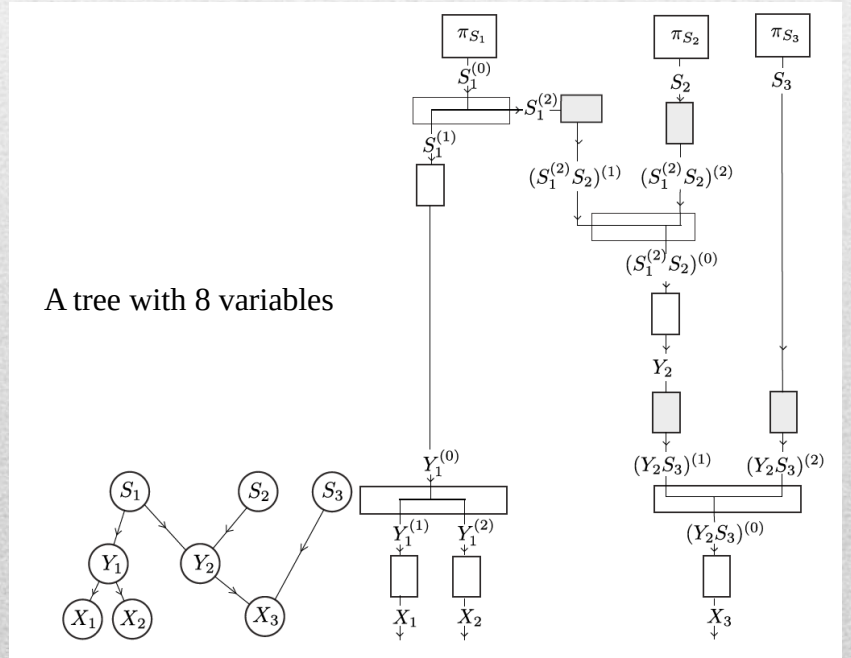




Three parents and a child



HMM



A tree with 8 variables

A numerical example:

$C = A + B$ (arithmetic sum); $A \in \{0, 1\}$; $B \in \{0, 1\}$; $C \in \{0, 1, 2\}$ (deterministic function)

$$P_1 = \tfrac{2}{4} I_2 \otimes 1_2^T = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix};$$

$$P_2 = \tfrac{2}{4} 1_2^T \otimes I_2 = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

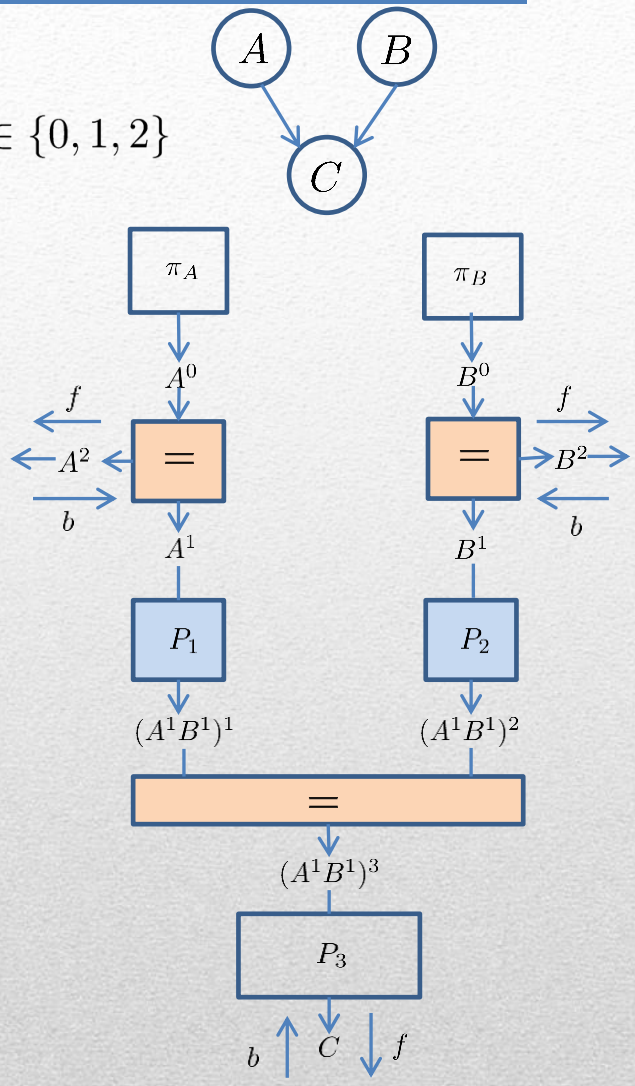
$$P_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix};$$

INPUTS: $b_{A^2} = \begin{bmatrix} .1 \\ .9 \end{bmatrix}$; $b_{B^2} = \begin{bmatrix} .99 \\ .01 \end{bmatrix}$; $b_C = \begin{bmatrix} .33 \\ .33 \\ .33 \end{bmatrix}$;

OUTPUTS: $f_{A^2} = \begin{bmatrix} .5 \\ .5 \end{bmatrix}$; $f_{B^2} = \begin{bmatrix} .5 \\ .5 \end{bmatrix}$; $f_C = \begin{bmatrix} .01 \\ .98 \\ .01 \end{bmatrix}$;

INPUTS: $b_{A^2} = \begin{bmatrix} .5 \\ .5 \end{bmatrix}$; $b_{B^2} = \begin{bmatrix} .99 \\ .01 \end{bmatrix}$; $b_C = \begin{bmatrix} .2 \\ .8 \\ .0 \end{bmatrix}$;

OUTPUTS: $f_{A^2} = \begin{bmatrix} .21 \\ .79 \end{bmatrix}$; $f_{B^2} = \begin{bmatrix} .56 \\ .45 \end{bmatrix}$; $f_C = \begin{bmatrix} .5 \\ .5 \\ .0 \end{bmatrix}$;



Issues:

1. Posterior calculation on trees is exact

(Pearl, 1988), (Lauritzen, 1996), (Jordan, 1998), (Loeliger, 2004), (Forney, 2001), (Bishop, 2006), (Barber, 2012), ….

……expressive power of trees if often limited

2. "Loopy graphs" (Chertkov, Chernyak and Teodorescu, 2008), (Murphy, Weiss, and Jordan, 1999), (Yedidia, Freeman and Weiss, 2000, 2005), (Weiss, 2000), (Weiss and Freeman, 2001)

…….simple belief propagation can lead to inconsistencies

Junction Trees (Lauritzen, 1996); **Cutset Conditioning** (Bidyuk and R. Dechter, 2007); **Monte Carlo sampling** (see for ex. Koller and Friedman, 2010); **Region method** (Yedidia, Freeman and Weiss, 2005).; **Tree Re-Weighted (TRW)** algorithm (Wainwright, Jaakkola and Willsky, 2005);

…….sometimes using simple loopy propagation gives good results if the loops are wide

3. Parameter learning

EM-learning: (Heckerman, 1996), (Koller and Friedman, 2010), (Ghahramani, 2012); **Variational Learning: (**Winn and Bishop, 2005)

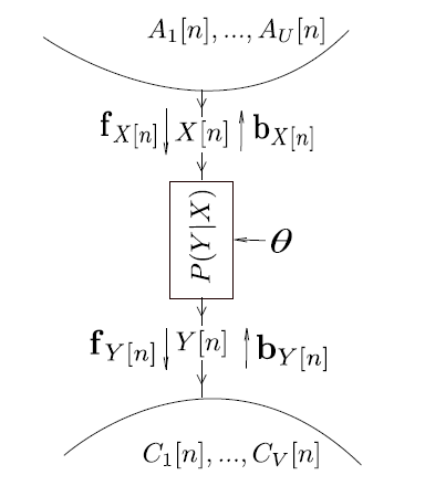
4. Structure Learning

Learning trees: (Chow and Liu, 1968) ,(Zhang, 2004), (Harmeling and Williams, 2011), (Palmieri, 2010), (Choi, Anandkumar and Willsky, 2011); **Learning general architectures (??)** (Koller and Friedman, 2010)

5. Applications

Coding; HMM; Complex scene analysis; Fusion of heterogeneous sources; ….opportunity of integrating more traditional signal processing with higher-levels of cognition!

Localized learning: (embedded)



- The factor graph in normal form reduces the system to one-in/one-out blocks
- Each block "sees" only local messages
- P(Y/X) is here a discrete-variable stochastic matrix
- EM approach on N training examples

$$P(XYA_1...A_UC_1...C_V;\theta) = P(C_1...C_V|Y)\ \underbrace{P(Y|X;\theta)}_{\text{to be learned}}\ P(XA_1...A_U).$$

$$L(\theta) = \prod_{n=1}^{N} \sum_x \sum_y p_{X[n]Y[n]\mathcal{E}[n]}(xy;\theta) = \prod_{n=1}^{N} \sum_x \sum_y f'_{X[n]}(x) p_{Y|X}(y|x;\theta) b'_{Y[n]}(y),$$

$$\ell(\theta) = \log(L(\theta)) = \sum_{n=1}^{N} \log\left(\mathbf{f}_{X[n]}^T\ \theta\ \mathbf{b}_{Y[n]}\right) + \sum_{n=1}^{N} \log\left(K_{f_{X[n]}} K_{b_{Y[n]}}\right)$$

$$\begin{cases} \min_\theta & -\sum_{n=1}^{N} L[n] \log\left(\mathbf{f}_{X[n]}^T\ \theta\ \mathbf{b}_{Y[n]}\right), \\ \theta & \text{row} - \text{stochastic}, \end{cases} \qquad \begin{cases} \min_\theta & \sum_{n=1}^{N} L[n] \sum_{i=1}^{M_y} b_{Y[n]}(j) \log \frac{b_{Y[n]}(j)}{\sum_{i=1}^{M_X} \theta_{ij} f_X[n](i)}. \\ \theta & \text{row} - \text{stochastic}, \end{cases}$$

ML learning **Minimum KL-divergence learning**

EM learning:

ML Algorithm:

(1) $\theta_{lm} \longleftarrow \frac{\theta_{lm}}{\sum_{n=1}^{N} L[n] f_{X[n]}(l)} \sum_{n=1}^{N} L[n] \frac{f_{X[n]}(l) b_{Y[n]}(m)}{\mathbf{f}_{X[n]}^{T} \theta \mathbf{b}_{Y[n]}}$,

(2) Row-normalize θ and back to (1)

KL Algorithm:

(1) $\theta_{lm} \longleftarrow \frac{\theta_{lm}}{\sum_{n=1}^{N} L[n] f_{X[n]}(l)} \sum_{n=1}^{N} L[n] \frac{f_{X[n]}(l) b_{Y[n]}(m)}{\sum_{i=1}^{M_X} \theta_{im} f_{X[n]}(i)}$,

(2) Row-normalize θ and back to (1).

VITERBI-like Algorithm:

(1) $\mathbf{e}_{X[n]} = I_{Max}(\mathbf{f_{X[n]}}) + \delta \mathbf{1}_{M_X \times 1}$;
$\mathbf{e}_{Y[n]} = I_{Max}(\mathbf{b_{Y[n]}}) + \delta \mathbf{1}_{M_Y \times 1}$;

(2) $\theta = \sum_{n=1}^{N} L[n] \mathbf{e}_{X[n]} \mathbf{e}_{Y[n]}^{T}$;

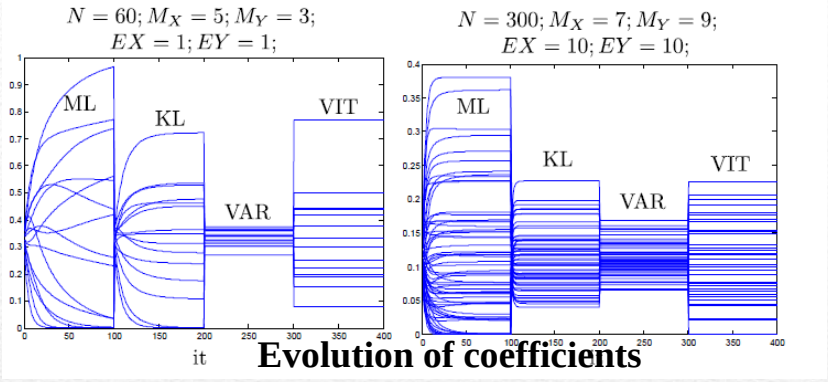
(3) Row-normalize θ.

VARIATIONAL Algorithm:

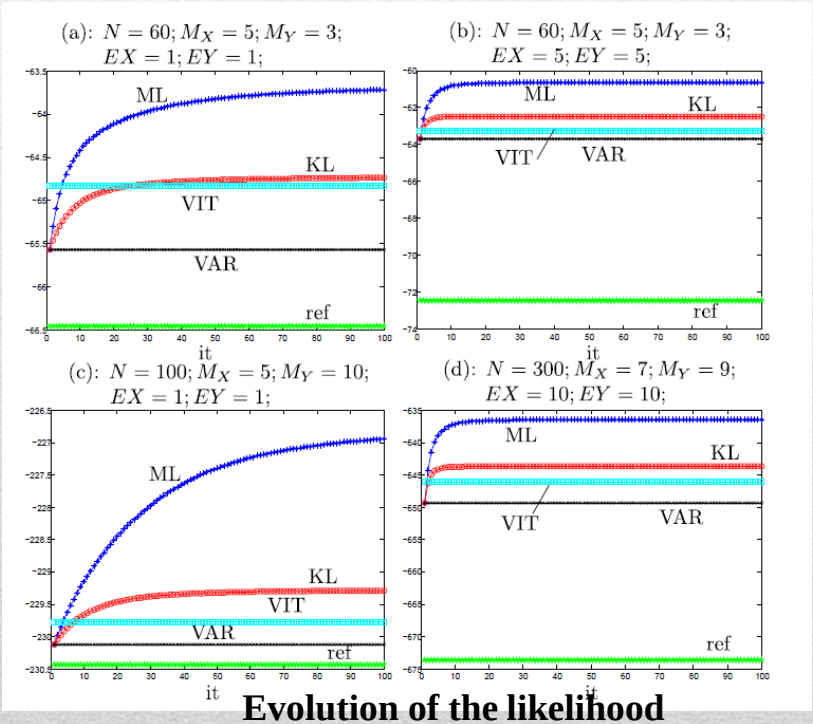
(1) $\theta_{lm} \longleftarrow \delta + \ sum_{n=1}^{N} L[n] f_{X[n]}(l) b_{Y[n]}(m)$,

(2) Row-normalize θ.

1. Simulations on a single block;
2. Varying sharpness ^E: 1-10
3. Similar behaviour for more complicated architectures
4. Greedy search: Local minima (multiple restarts)



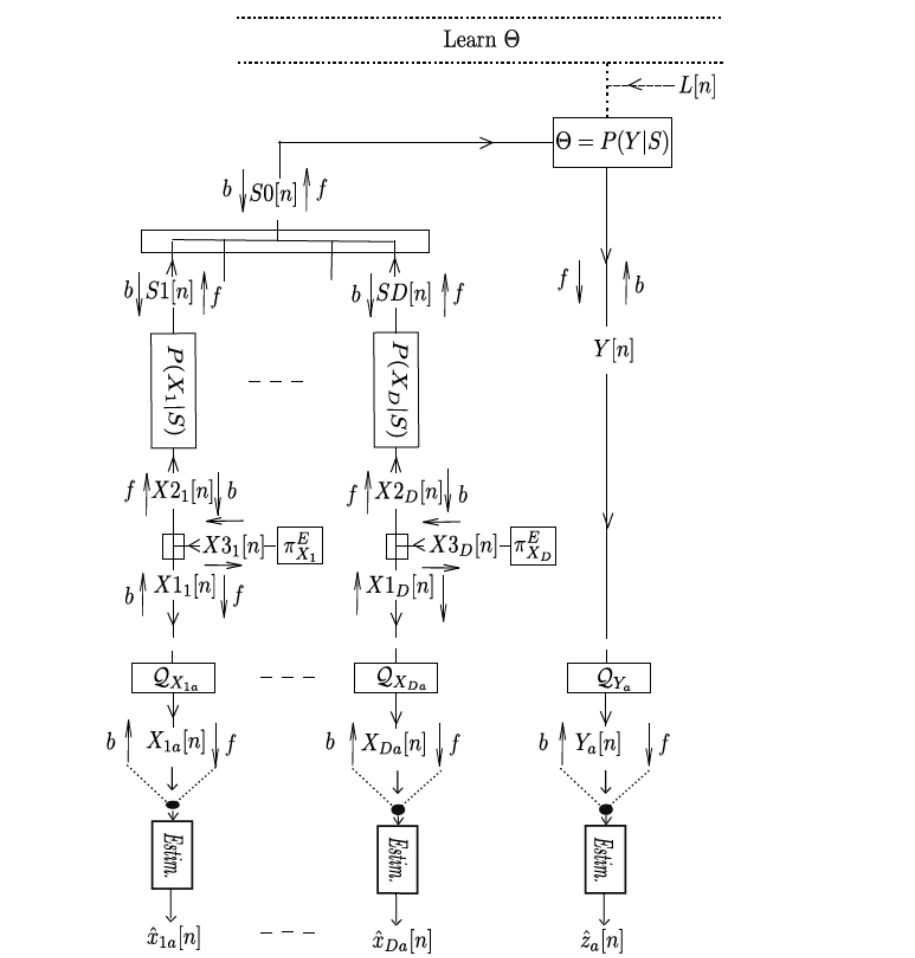
Evolution of coefficients



Evolution of the likelihood

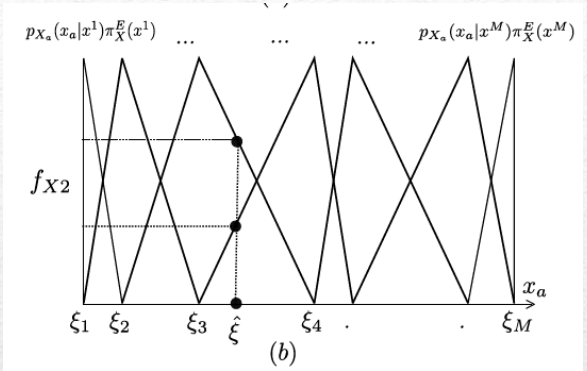
F. A. N. Palmieri, "A Comparison of Algorithms for Learning Hidden Variables in Normal Graphs", *submitted for journal publication, Jan 2014,* arXiv: 1308.5576v1 [stat.ML]

Application 1: Learning a Nonlinear Function



The factor graph for learning the D-dimensional function $Y_a = g(X_{1a}, ..., X_{Da})$.

1. Soft quantization/dequantization (triangular likelihoods with entropic priors)



2. Map input variables to an embedding space

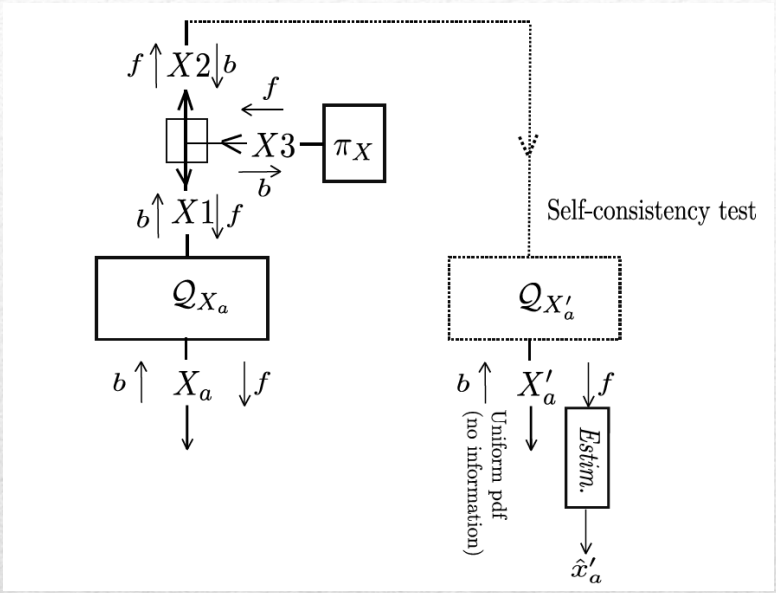
3. Minimize $KL(bY||fY)$

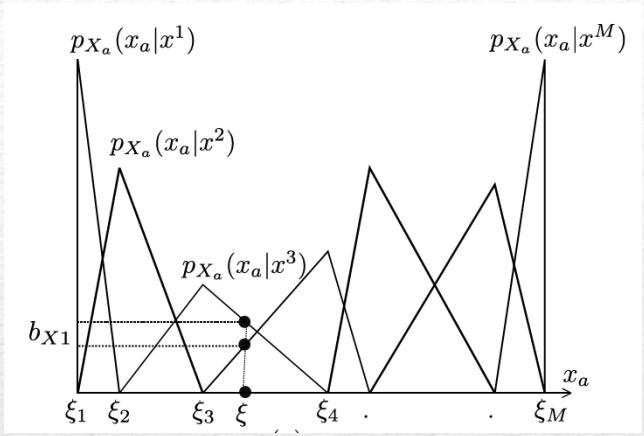
- Not to challenge techniques for nonlinear adaptive filters (SVM, NN, RBF,..);
- Provide a technique for fusing categorical discrete data into a unique framework;
- Numerous applications in signal processing

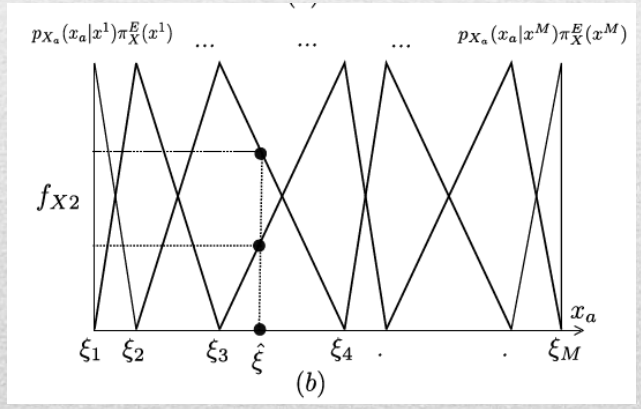
Francesco A. N. Palmieri, "Learning Non-Linear Functions with Factor Graphs," *IEEE Transactions on Signal Processing, Vol.61, N. 17, pp.* 4360 - 4371, 2013.

Application 1: Learning a Nonlinear Function (cont.)

Bidirectional quantizer







$$\pi_X^E \propto \left(e^{H(X|1)}, e^{H(X|2)}, ..., e^{H(X|M)}\right)$$

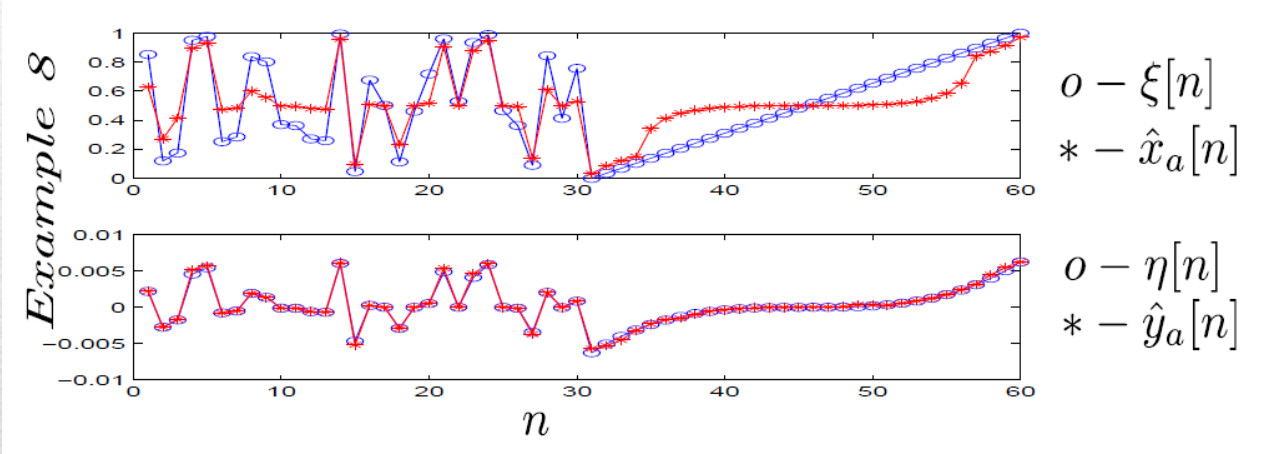
Entropic priors

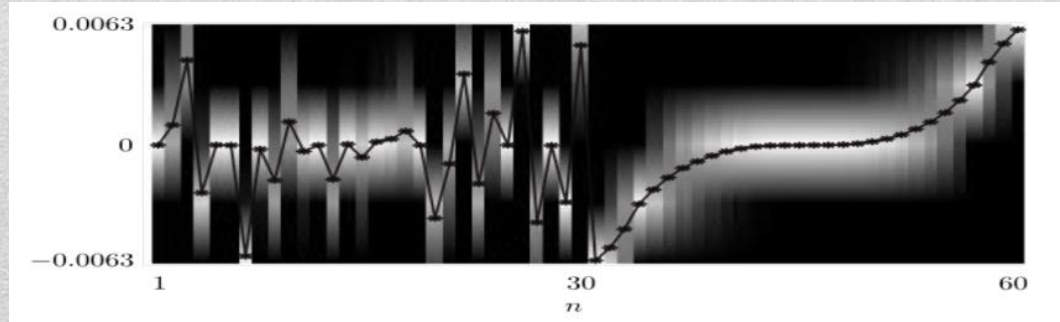
Francesco A. N. Palmieri and Domenico Ciuonzo, "Objective Priors from Maximum Entropy in Data Classification," *Information Fusion*, February 14, 2012,

Application 1: Learning a Nonlinear function (cont.)

$Y = \frac{1}{20}(X - \frac{1}{2})^3;\ X \in [0, 1];\ M_X = 20,\ M_Y = 5;$

- Difficulties with the inverse map in the flat area (very small first derivative).



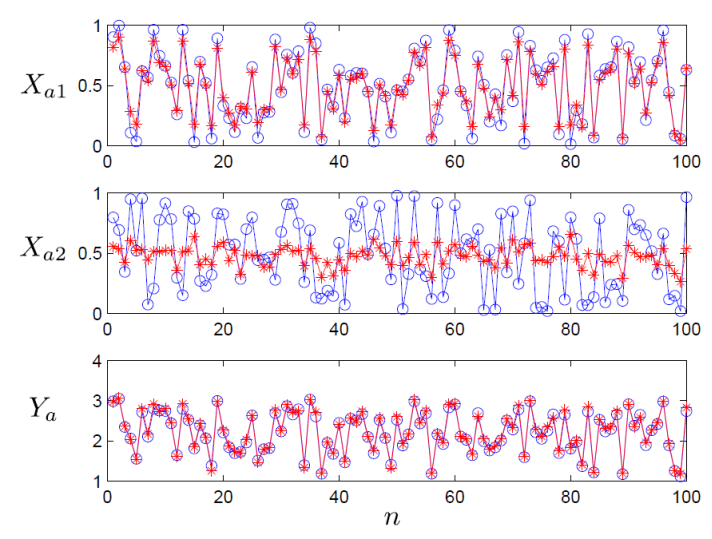


grayscale picture of $f_{Y_a[n]}$
(estimated points superimposed)

Application 1: Learning a Nonlinear function (cont.)

$Y_a = \sqrt{X_{a1}^2 + X_{a2}^2} + \sin X_{a1} + 1,\ M_1 = M_2 = M_Y = 4$

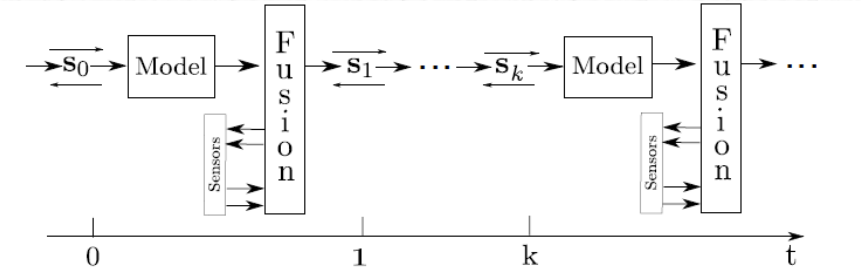
$N = 100$ random independent pairs from $[0, 1]^2$ for X_{a1} and X_{a2}. $M_1 = M_2 = M_Y = 4$.

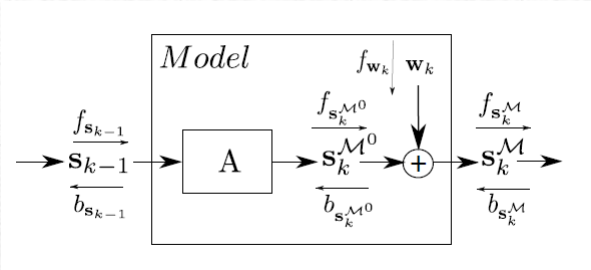


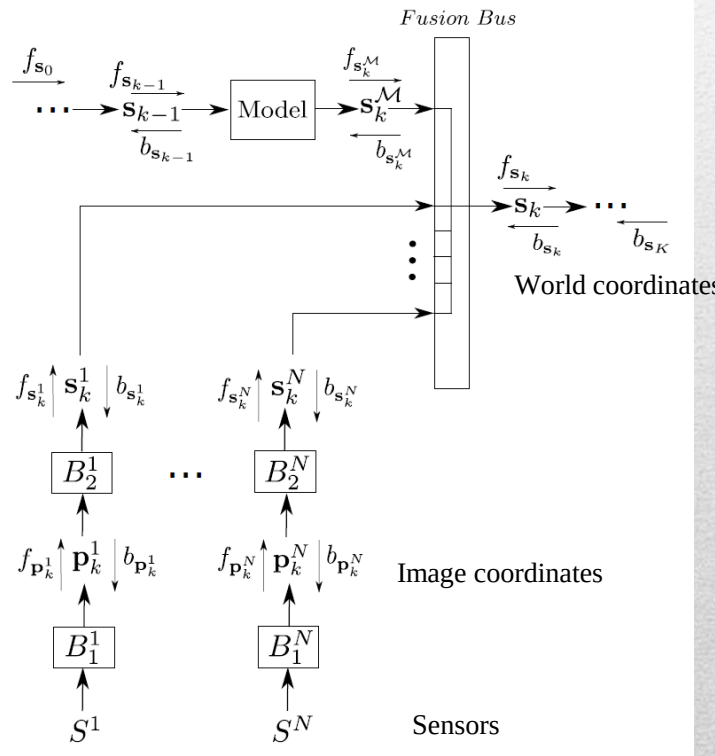
0 - backward

* - forward

Application 2: Tracking objects with cameras







Gaussian messages (means and covariances):

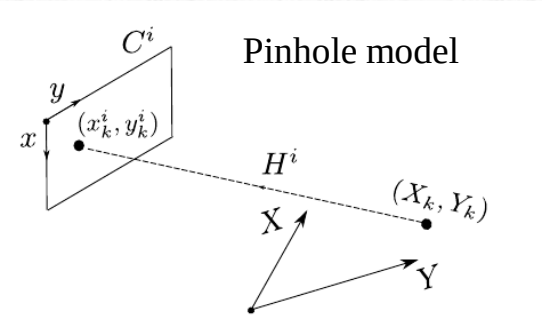
$$f_{\mathbf{s}_k^{\mathcal{M}^0}} = \{Am_{f_{\mathbf{s}_{k-1}}}, A\Sigma_{f_{\mathbf{s}_{k-1}}}A^T\},$$
$$f_{\mathbf{s}_k^{\mathcal{M}}} = \{m_{f_{\mathbf{s}_k^{\mathcal{M}^0}}} + m_{f_{\mathbf{w}_k}}, \Sigma_{f_{\mathbf{s}_k^{\mathcal{M}^0}}} + \Sigma_{f_{\mathbf{w}_k}}\},$$
$$b_{\mathbf{s}_k^{\mathcal{M}^0}} = \{m_{b_{\mathbf{s}_k^{\mathcal{M}}}} - m_{f_{\mathbf{w}_k}}, \Sigma_{b_{\mathbf{s}_k^{\mathcal{M}}}} + \Sigma_{f_{\mathbf{w}_k}}\},$$
$$b_{\mathbf{s}_{k-1}} = \{(A^T\Sigma^{-1}_{b_{\mathbf{s}_k^{\mathcal{M}^0}}}A)^{-1}A^T\Sigma^{-1}_{b_{\mathbf{s}_k^{\mathcal{M}^0}}}m_{b_{\mathbf{s}_k^{\mathcal{M}^0}}}, (A^T\Sigma^{-1}_{b_{\mathbf{s}_k^{\mathcal{M}^0}}}A)^{-1}\}.$$

$$f_{\mathbf{s}_k} = f_{\mathbf{s}_k^{\mathcal{M}}} \odot (\odot_{j=1}^{N} f_{\mathbf{s}_k^j}),$$
$$b_{\mathbf{s}_k^{\mathcal{M}}} = b_{\mathbf{s}_k} \odot (\odot_{j=1}^{N} f_{\mathbf{s}_k^j}),$$
$$b_{\mathbf{s}_k^i} = b_{\mathbf{s}_k} \odot f_{\mathbf{s}_k^{\mathcal{M}}} \odot (\odot_{j=1,j\neq i}^{N} f_{\mathbf{s}_k^j}),\ i = 1, ..., N.$$

(Kalman filter equations ‘’pipelined’’)

H.-A. Loeliger, J. Dauwels, J. Hu, S. Korl, L. Ping, and F. Kschischang, The factor graph approach to model-based signal processing, *Proceedings of the IEEE*, vol. 95, no. 6, pp. 12951322, 2007.

Application 2: Tracking objects with cameras (cont.)



World coordinates ⟵ Image coordinates

$$\begin{pmatrix} X_k/\lambda_k^i \\ Y_k/\lambda_k^i \\ 1/\lambda_k^i \end{pmatrix} = R^i \begin{pmatrix} x_k^i \\ y_k^i \\ 1 \end{pmatrix},$$

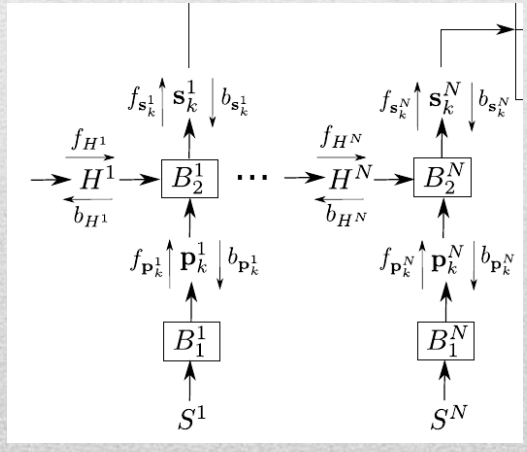
$$X_k = \frac{X_k/\lambda_k^i}{1/\lambda_k^i} = \frac{r_{11}^i x_k^i + r_{12}^i y_k^i + r_{13}^i}{r_{31}^i x_k^i + r_{32}^i y_k^i + r_{33}^i} = g_1^i(x_k^i, y_k^i),$$
$$Y_k = \frac{Y_k/\lambda_k^i}{1/\lambda_k^i} = \frac{r_{21}^i x_k^i + r_{22}^i y_k^i + r_{23}^i}{r_{31}^i x_k^i + r_{32}^i y_k^i + r_{33}^i} = g_2^i(x_k^i, y_k^i),$$
$$\dot{X}_k = \frac{dX_k^i}{dt} = g_3^i(x_k^i, y_k^i, \dot{x}_k^i, \dot{y}_k^i),$$
$$\dot{Y}_k = \frac{dY_k^i}{dt} = g_4^i(x_k^i, y_k^i, \dot{x}_k^i, \dot{y}_k^i),$$

Image coordinates ⟵ World coordinates

$$\lambda_k^i \begin{pmatrix} x_k^i \\ y_k^i \\ 1 \end{pmatrix} = H^i \begin{pmatrix} X_k \\ Y_k \\ 1 \end{pmatrix},$$

Homography matrix
(learned from calibration points)

$$x_k^i = \frac{\lambda_k^i x_k^i}{\lambda_k^i} = \frac{h_{11}^i X_k + h_{12}^i Y_k + h_{13}^i}{h_{31}^i X_k + h_{32}^i Y_k + h_{33}^i} = q_1^i(X_k, Y_k),$$
$$y_k^i = \frac{\lambda_k^i y_k^i}{\lambda_k^i} = \frac{h_{21}^i X_k + h_{22}^i Y_k + h_{23}^i}{h_{31}^i X_k + h_{32}^i Y_k + h_{33}^i} = q_2^i(X_k, Y_k),$$
$$\dot{x}_k^i = \frac{dx_k^i}{dt} = q_3^i(X_k, Y_k, \dot{X}_k, \dot{Y}_k),$$
$$\dot{y}_k^i = \frac{dy_k^i}{dt} = q_4^i(X_k, Y_k, \dot{X}_k, \dot{Y}_k).$$



- Local first-order approximations for Gaussian pdf propagation;
- Gaussian noise on the homography matrix

Application 2: Tracking objects with cameras (cont.)

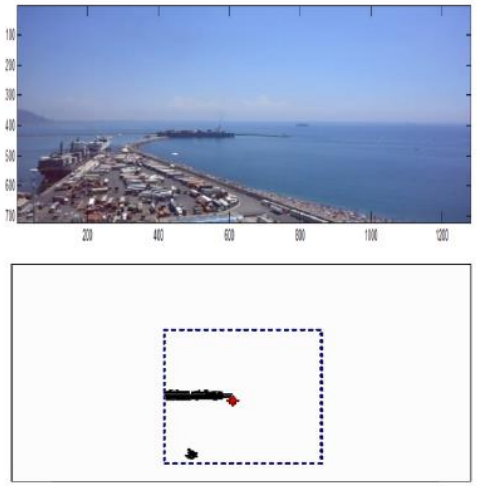
Salerno (Italy) harbour (3 commercial cameras)

Typical views

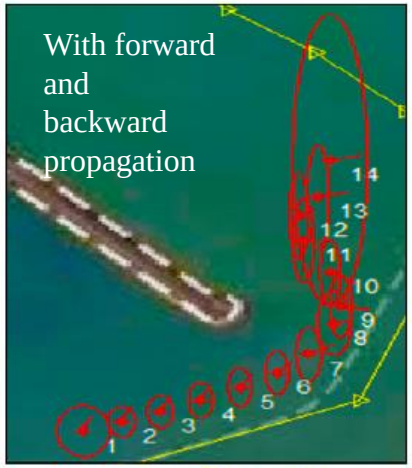
Francesco Castaldo and Francesco A. N. Palmieri, ‘’Image Fusion for Object Tracking Using Factor Graphs,’’ *Proc. of IEEE-AES Conference*, Montana, March 2-7, 2014.

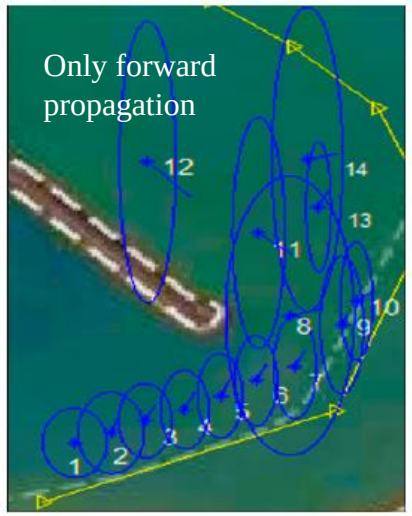
F. Castaldo and F. A. N. Palmieri, "Target Tracking using Factor Graphs and Multi-Camera Systems,“ submitted, Jan 2014.

Application 2: Tracking objects with cameras (cont.)

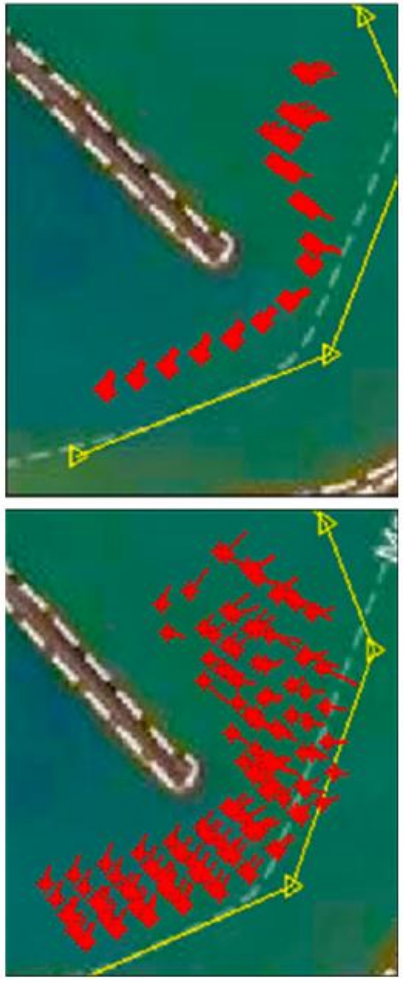


Background subtraction algorithm





No calibration error
(covariances amplified 10^6)



With calibration error
(10^-3; 10^-4)

Application 3: Multi-layer convolution graphs

- Striking achievements in "deep belief networks" rely on convolutional and recurrent structures in multi-layer neural networks (Hinton, Le Cun, Bengio, Ng)
- Convolutive paradigms in Bayesian factor graphs?
- Convolutive structures better than trees account for short distance chained dependences;
- Expansion to hierarchies to capture long-term dependence at a gradually increasing scale.

triplets

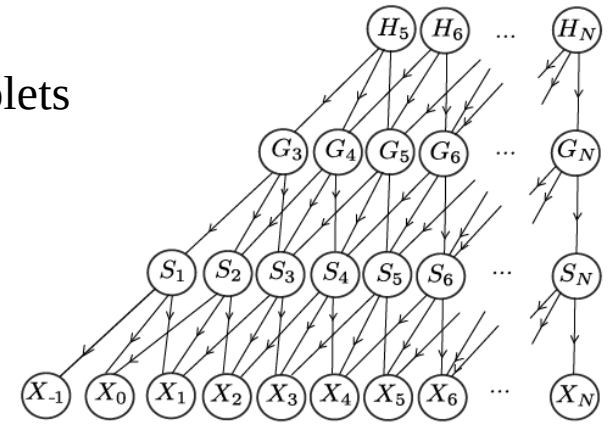


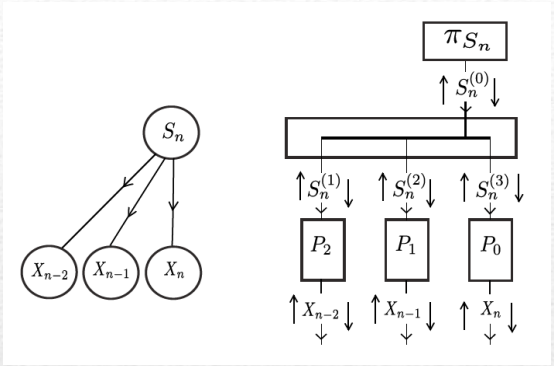
Fig. 1. Example of three-layer convolution graph with overlap $M = 2$ on all layers.

Many many loops!!

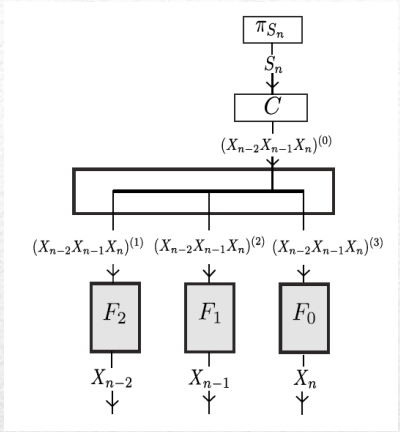
It appears intractable for message propagation;

Stationarity allows a transformation

Application 3: Multi-layer convolution graph (cont.)

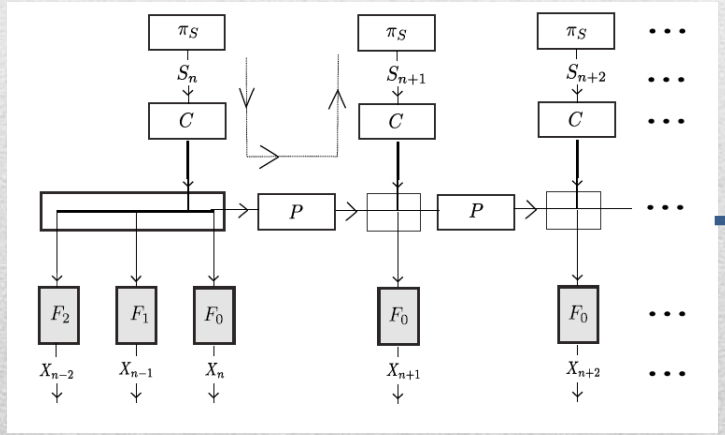


Latent model



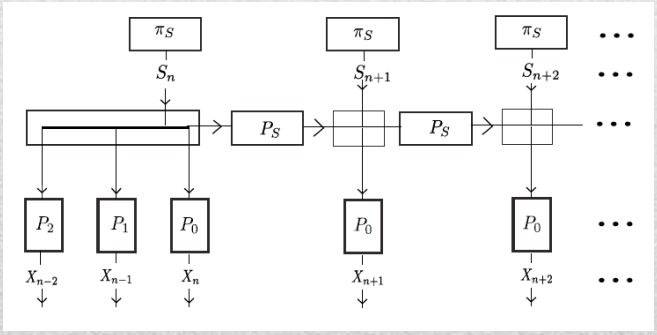
$CF_1 \simeq P_1;$
$CF_2 \simeq P_2;$
$CF_3 \simeq P_3;$
C row-stochastic

Explicit mapping to product space



Junction tree

$$P = P(X_{n-1}X_nX_{n+1}|X_{n-2}X_{n-1}X_n) = \mathbf{1}_d \otimes I_d \otimes I_d \otimes \tfrac{1}{d}\mathbf{1}_d^T.$$



HMM approximation

$P_S = CPC^T;$

Learned from data

if the product space is too large

Applicaton 3: Multi-layer convolution graph (cont.)

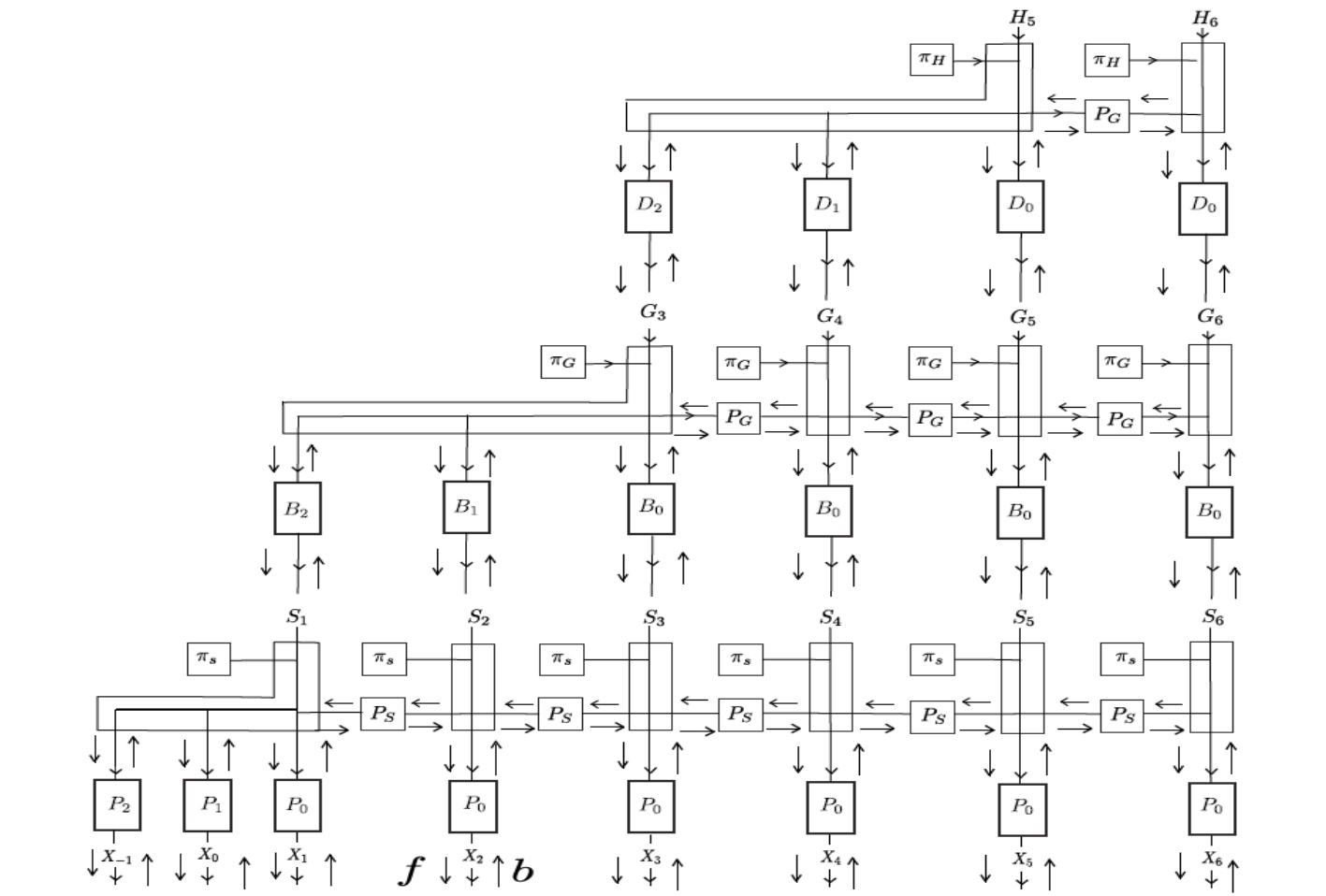
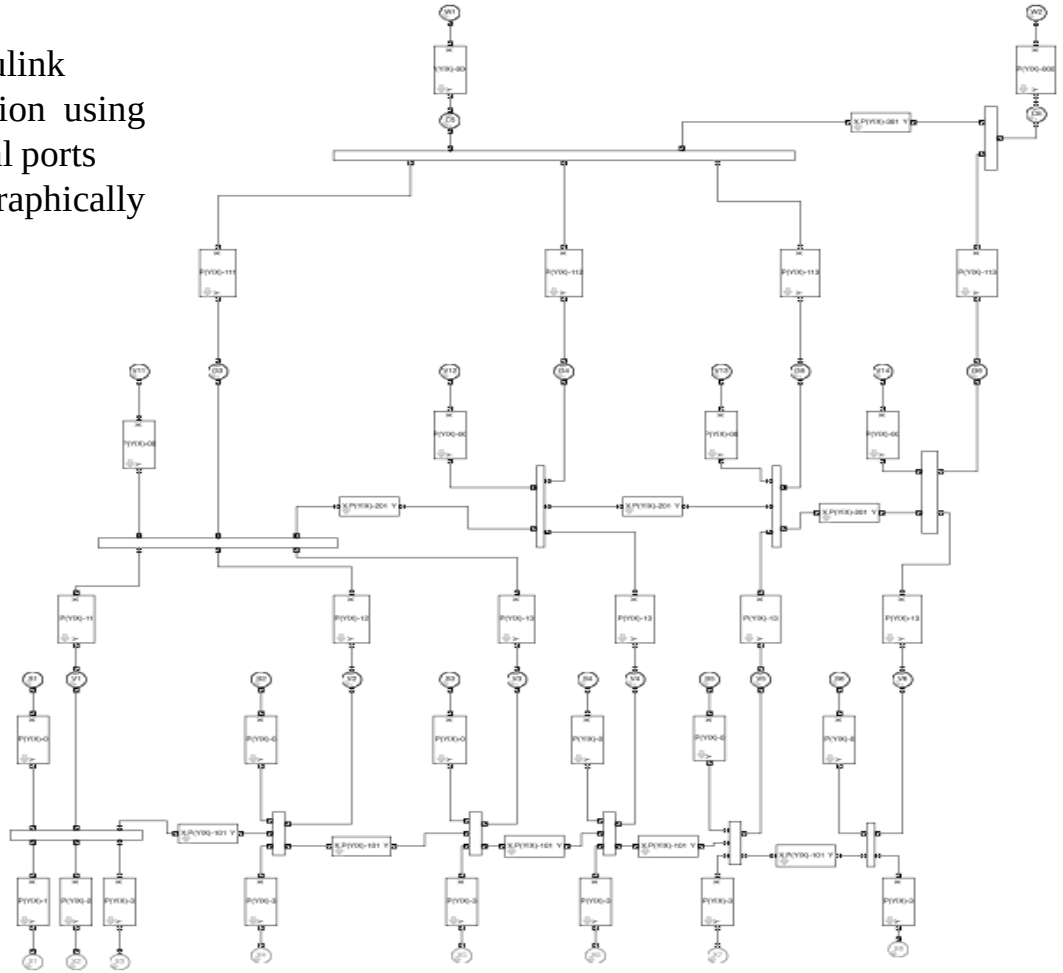


Fig. 6. The three-layer normal graph approximating the convolution graph of Figure 1 for $N = 6$.

Applicaton 3: Multi-layer convolution graph (cont.)

Matlab/Simulink implementation using bi-directional ports assembled graphically



Applicaton 3: Multi-layer convolution graph (cont.)

Simulation on controlled environment and limited computational complexity:

-$d = 27$ character set from The Waste Land by T. S. Eliot:

i think we are in rats alley where the dead men lost their bones

- Used HMM approximation

- EM algorithm and manual the triplets from the text.

- Embedding variables thave sizes $M_S = 59$, $M_G = 100$ and $M_H = 100$.

- Belief propagation in the system is run for 30 steps In learning we have used 10 epoques and the greedy ML algorithm

(1) present a set of 8 characters X_{-2} through X_6 to the bottom;
(2) perform belief propagation on the first layer with no connections to the second layer;
(3) collect the forward messages for S_1 through S_6 and use them to learn the latent model on triplets for the second layer;
(4) back to (1) with a new set of 8 characters at the bottom by sliding on the text.

- Tested the system at X_{-2}, X_{-1},...,X_6 for various layer configurations by injecting our queries through the backward messages and collecting the results as the forward messages.

Applicaton 3: Multi-layer convolution graph (cont.)

i think we are in rats alley where
the dead men lost their bones

Incomplete input: **re~the??**
one- and two-layer graph, one error: **re~their**
three-layer graph, no error: **re~the~d**

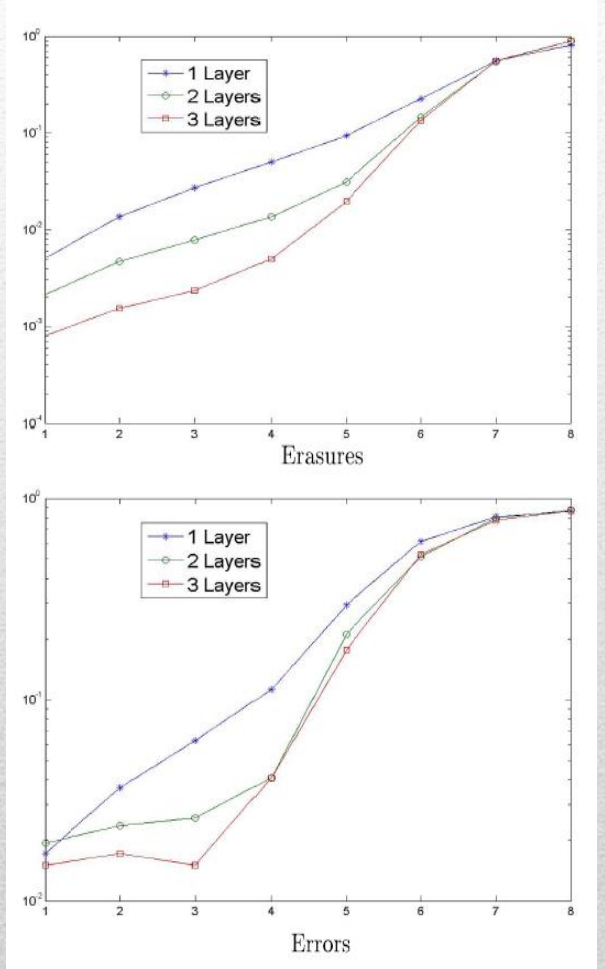
Incomplete input: **o~~~the?**
one- and two- layers: **ost~the~**
even if in the two-layer response there is an equal maximum probability on both **~** and **i**
three-layers increase the probability on **i**

Wrong input: **re~tke~m**
One- two-layers, errors; three layers, no error: **re~the~d**

Input: **lbeherde**
one- two-layers, errors; three-layers, no error: **e~the~de**

Arbitrary input: **asteland**
three-layers (getting closer to the dataset): **k~we~are**

----→Extension to Larger datasets and images

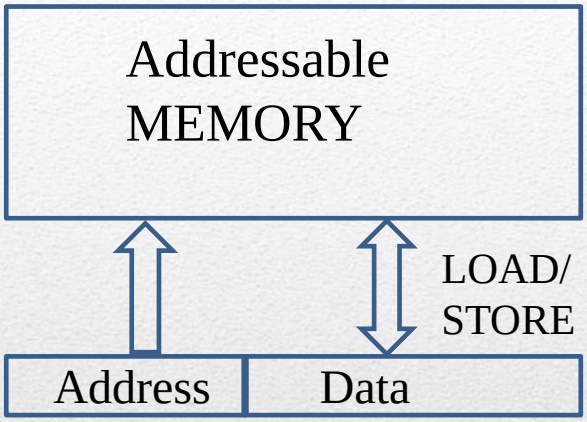


Probabilistic computers ???

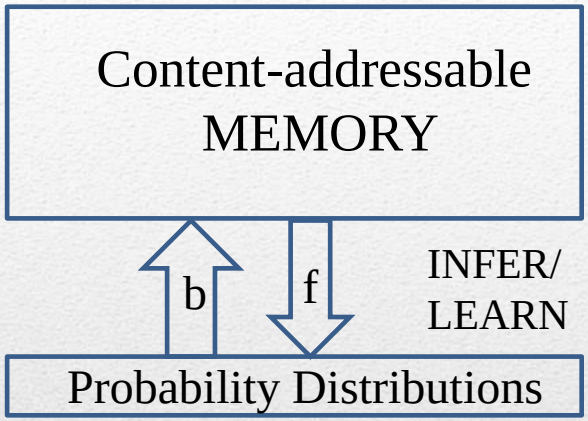
- Very consistent results on inference and learning with Bayesian networks;
- Many successful applications are based on Bayesian paradigms;
- Will the probability pipelines scale in complexity?
- New architectures/languages that include uncertainties?

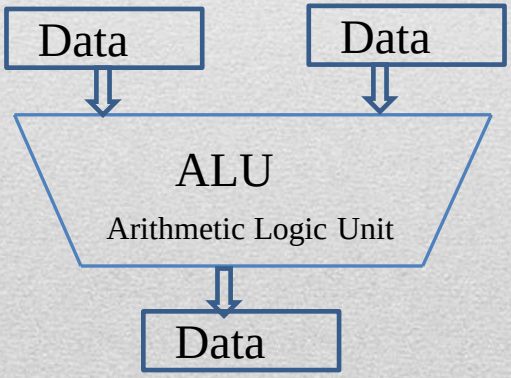
Probabilistic computers ???

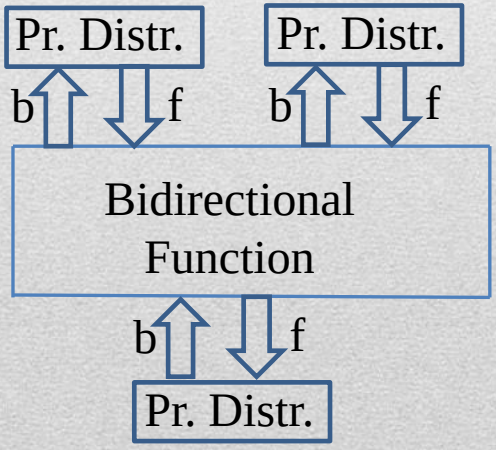
Traditional architecture



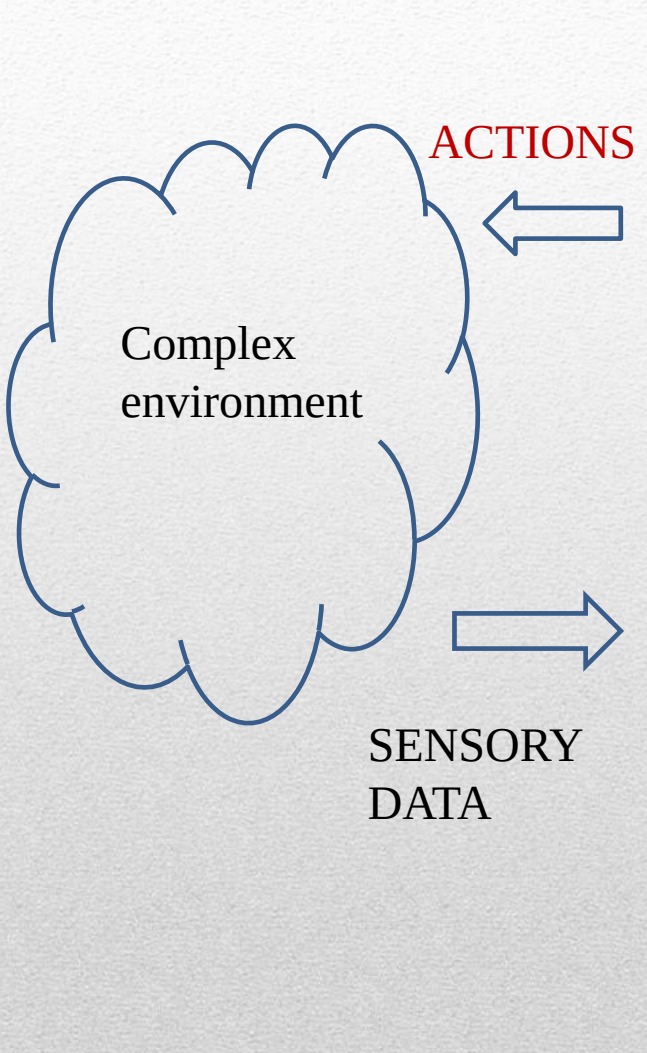
Probabilistic architecture (?!)



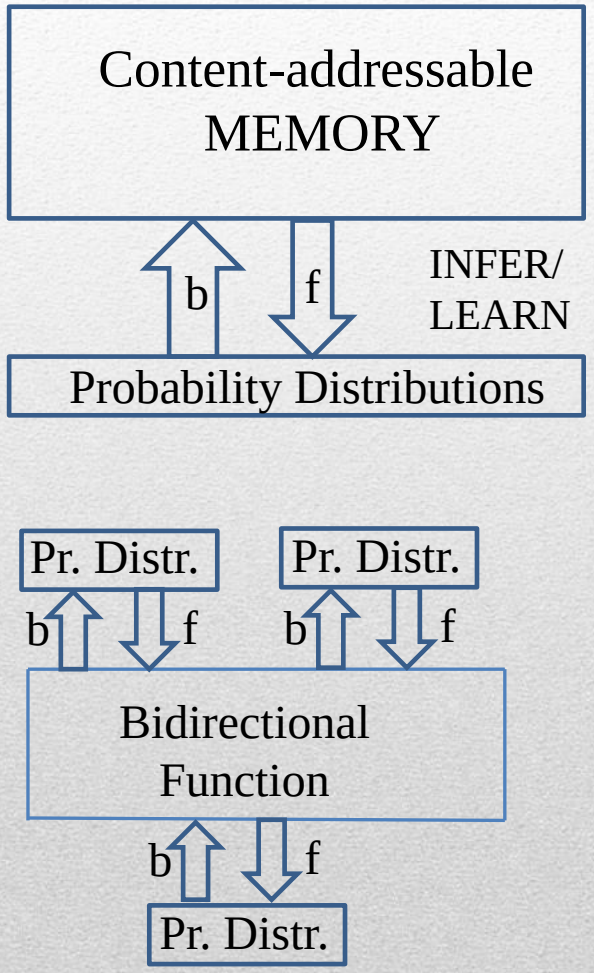




Probabilistic computers ???



Probabilistic architecture (?!)



Conclusions and future directions:

The Bayesian framework is effective in a number of signal processing applications

Beyond hard logic

Bidirectional probability propagation shows promising impact on the applications

Complexity scale in dealing with unstructured environments

Probabilistic computers

Extensions of signal processing to (stochastic) control of action in integration with uncertainty

Thanks for your kind attention.

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