

# **Bistable and Adaptive Piezoelectric Circuitry for Impedance-Based Structural Health Monitoring**

Jinki Kim

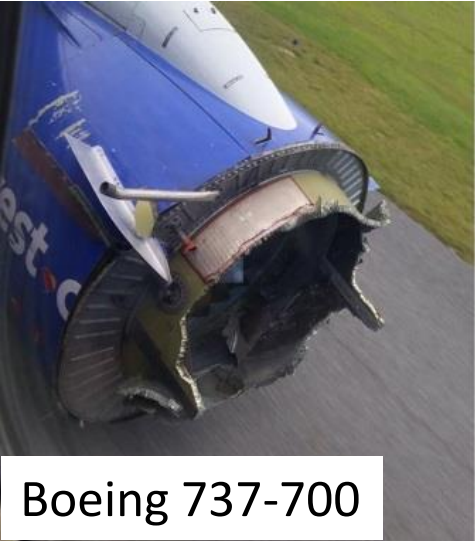
Department of Mechanical Engineering  
University of Michigan

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# Outline of the Presentation

- **Background on Structural Health Monitoring**
- **Problem Statement and Research Goal**
- **Impedance Data Enrichment Concept**
- **Bifurcation-based Sensing Method**
- **Scholarly Contributions and Impacts**
- **Proposal for Future Plan**

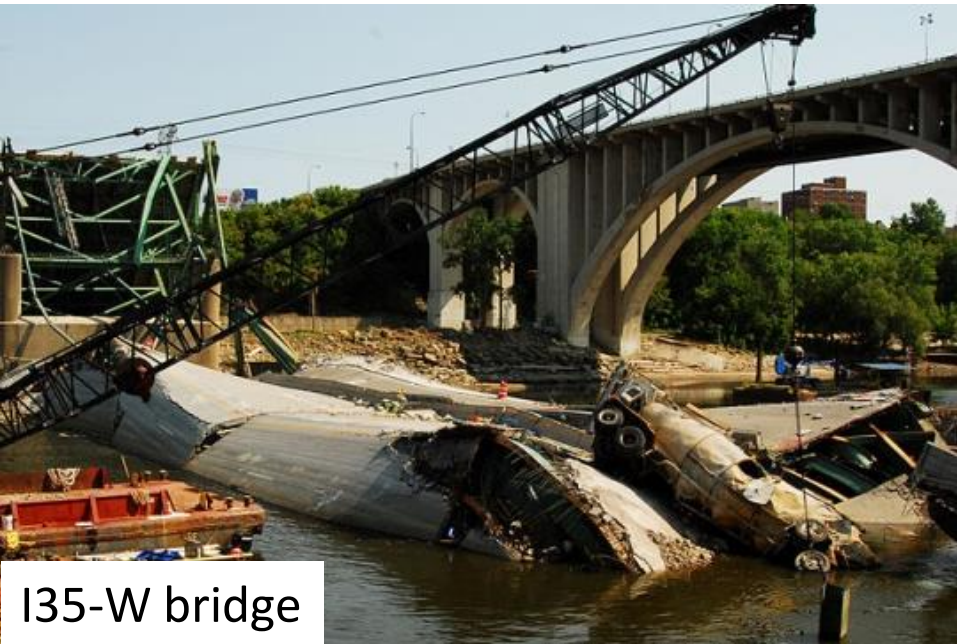
# Background: Structural Health Monitoring



Boeing 737-700



Ocotillo CA



I35-W bridge

# Background: Structural Health Monitoring



**System Health Management:** *Capabilities of a system that preserve the system's ability to function as intended*

→ Life-safety and economic impact

## Diagnosis

Structural Health Monitoring (SHM)

Usage Monitoring

## Prognosis

System Health Management

## Determines

- foundation location,
- maintenance
- useful life
- operational plan

# Background: Structural Health Monitoring

## Vibration-based methods (Farrar et al 2001; Carden and Fanning, 2004)

- **Merit:** Easy to implement, large sensing area  
Identification of damage location and severity
- **Limitation:** Low frequency / not sensitive to small damage

## Wave propagation-based methods (Raghavan and Cesnik 2007)

- **Merit:** High frequency / incipient small damage detectable
- **Disadvantage:** Challenges in identifying the severity of damage

## Piezoelectric impedance-based methods (Park et al 2003; Wang and Tang 2010)

- Utilize the electromechanical coupling effect of the piezoelectric transducer
- **Simple implementation**
- **Damage location / severity identification**
- **High sensitivity to small damages**

$$i\omega C_p \left( 1 - \nu_{31} \frac{\hat{Z}_{str}(\omega)}{\hat{Z}_p(\omega)} \right)$$

Piezoelectric impedance

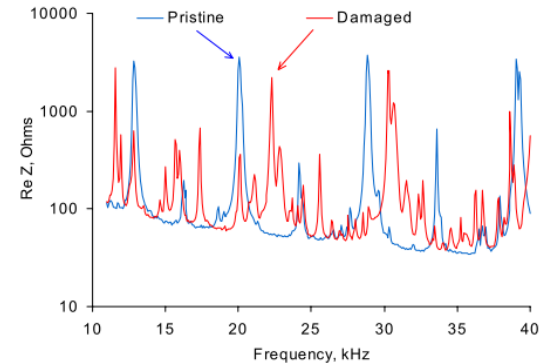
Structure



# Background: Impedance-based Methods

## Impedance data-based methods

- Frequency spectra analysis methods
  - **Merit:** straightforward implementation
  - **Limitation:** only detection  
non-physical damage index



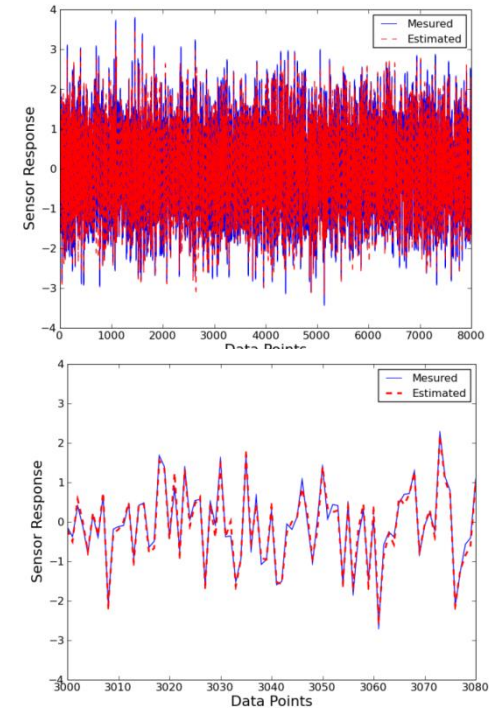
Zagrai, Giurgiutiu, 2002

## Impedance model-based methods

# Background: Impedance-based Methods

## Impedance data-based methods

- **Frequency spectra analysis methods**
  - **Merit:** straightforward implementation
  - **Limitation:** only detection  
non-physical damage index
- **Time-series analysis methods**
  - **Merit:** real-time health monitoring
  - **Limitation:** only detection  
phenomenological model



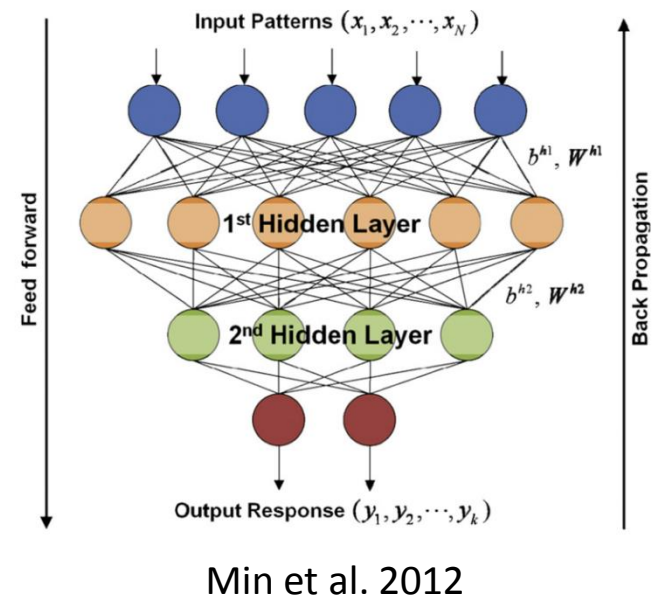
Park et al. 2010

## Impedance model-based methods

# Background: Impedance-based Methods

## Impedance data-based methods

- **Frequency spectra analysis methods**
  - **Merit:** straightforward implementation
  - **Limitation:** only detection  
non-physical damage index
- **Time-series analysis methods**
  - **Merit:** real-time health monitoring
  - **Limitation:** only detection  
phenomenological model
- **Feature-based pattern recognition methods**
  - **Merit:** identification of damage location/severity
  - **Limitation:** large training data required for all different damage cases



## Impedance model-based methods



# Background: Impedance-based Methods

## Impedance data-based methods

### Impedance model-based methods

- These methods are based on *physical model* → *inverse problem: identify system parameters (damage) of model based on measurements*
- **Merit:** Damage location/severity identification for new damage condition
  - Sensor self-diagnosis,
  - Optimize sensor deployment strategy
- **Various approaches**
  - FEM-based model updating (Tseng, et.al. 2005)
  - Spectral Element Method (SEM)-based model updating (Ritdumrongkul, et. al. 2005)
  - Sensitivity-based inverse problem using SEM (Wang, Tang. 2010)
- **Limitation:** *Seriously underdetermined* inverse problem
  - Little consideration on the *noise and uncertainty in modeling*

# Background: Underdetermined inverse problem

model                      damage index  
 $\mathbf{G} \times \delta \mathbf{d} = \delta \mathbf{Y}$  ← measurement

→ Moore-Penrose pseudoinverse

$$\begin{aligned}
 \delta \tilde{\mathbf{d}} &= \mathbf{G}^{-1} \delta \mathbf{Y} \\
 &= (\mathbf{V} \mathbf{S}^{-1} \mathbf{U}^T) \delta \mathbf{Y} \\
 &\approx \sum_{i=1}^n \frac{\mathbf{u}_i^T \delta \mathbf{Y}}{s_i} \mathbf{v}_i
 \end{aligned}$$

↑ truncate the singular values

$$\mathbf{G} \times \delta \mathbf{d} = \delta \mathbf{Y} + \mathbf{e}$$

Error ↓

$$\begin{aligned}
 \delta \mathbf{d}^* &= \mathbf{G}^{-1} (\delta \mathbf{Y} + \mathbf{e}) \\
 &= \delta \tilde{\mathbf{d}} + \mathbf{G}^{-1} \mathbf{e} \\
 &\approx \delta \tilde{\mathbf{d}} + \sum_{i=1}^n \frac{\mathbf{u}_i^T \mathbf{e}}{s_i} \mathbf{v}_i
 \end{aligned}$$

very small singular values amplify the error term!

Damage identification is **extremely sensitive to small errors** for underdetermined inverse problem.

# Problem Statement and Research Goal

## Problem Statement

- The inverse problem for damage identification is significantly underdetermined → Extremely sensitive to small errors such as environmental noise
- Accurate measurement of damage induced piezoelectric impedance variations, especially with noise

## Research Goal

To **overcome the limitations** and develop a new method that can accurately and completely capture the damage features from piezoelectric impedance variations while still **maintain the simplicity** of the approach

# Problem Statement and New Idea I

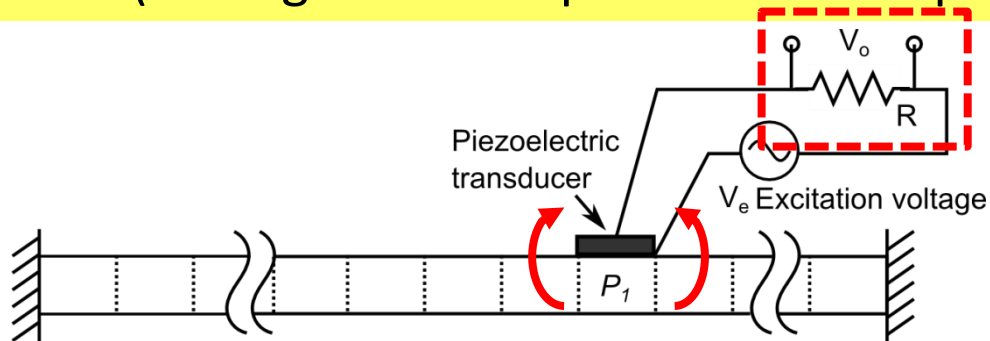
## Problem Statement

- The inverse problem for damage identification is significantly underdetermined → Extremely sensitive to small errors such as environmental noise
- Accurate measurement of damage induced piezoelectric impedance variations, especially with noise

**New Idea: Impedance Data Enrichment via Adaptive Piezoelectric Circuitry**

# Sensitivity-based Inverse Problem Formulation

Derive  $\delta \mathbf{d}$  (location and severity of damage, e.g., stiffness reduction) based on  $\delta \mathbf{Y}$  (damage-induced piezoelectric impedance variation) measurements



Generalized force by piezoelectric transducer

$$\mathbf{F} = M_p \Phi_p^T$$

Generalized displacement of the structure

$$\mathbf{W} = \mathbf{S}^{-1}(\omega) \mathbf{F}$$

Voltage generated by displacement

$$V_p = K_2 \Phi_p \mathbf{W}$$

Impedance measured by the voltage across R

$$\rightarrow \mathbf{Y} = \frac{1}{R} \frac{V_o}{V_e} = \frac{1}{R} K_2 [K_1 \Phi_p \mathbf{S}^{-1}(\omega) \Phi_p^T + 1]$$

**Cannot be accurately predicted!**

First-order sensitivity equation

$$\mathbf{G} \times \delta \mathbf{d} = \delta \mathbf{Y}$$

$\mathbf{G}$ :  $m \times N$  sensitivity matrix

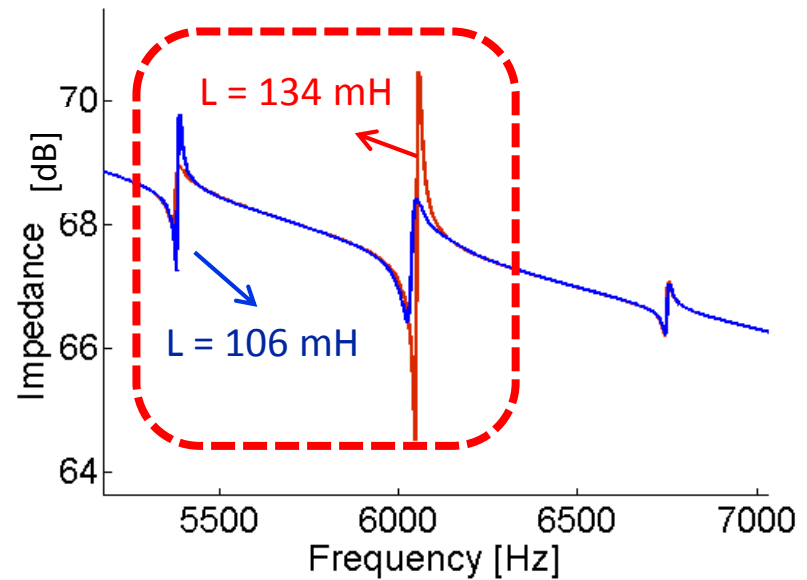
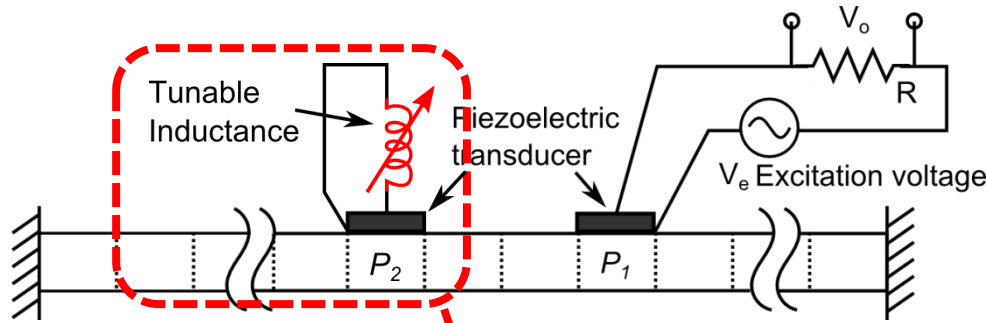
$\delta \mathbf{Y}$ :  $m \times 1$  vector of impedance variation *measurements*

$\delta \mathbf{d}$ :  $N \times 1$  vector of *damage index*

$$m \ll N$$

**→ Underdetermined!**

# New Idea: Impedance Data Enrichment via Adaptive Piezoelectric Circuitry



Generalized force by the additional piezoelectric transducer

$$\mathbf{F} = M_p \Phi_p^T$$

New dynamic stiffness matrix is adjustable via tuning inductance

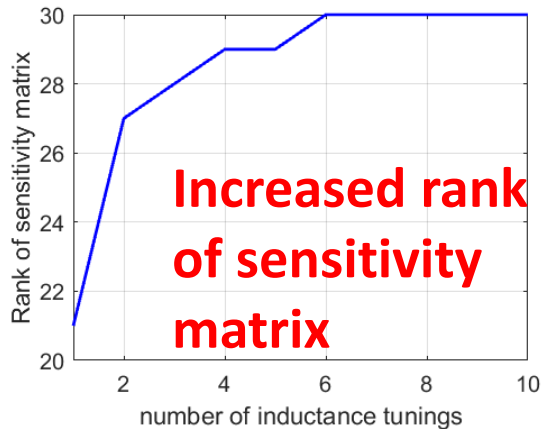
$$\mathbf{W} = \left( \mathbf{S} \left[ \quad \right] \right)^{-1} M_p \Phi_p^T$$

Tuning the inductance systematically can result in a **family** of impedance data

**Greatly enrich the impedance measurement data and gain more information about the structural damage**

# New Idea: Impedance Data Enrichment via Adaptive Piezoelectric Circuitry

Tune the inductance to form a sequence,  $L = [L_1, L_2, \dots, L_n]$



$$\begin{bmatrix} \mathbf{G}(L_1) \\ \mathbf{G}(L_2) \\ \vdots \\ \mathbf{G}(L_n) \end{bmatrix} \times \delta \mathbf{d} = \begin{Bmatrix} \delta \mathbf{Y}(L_1) \\ \delta \mathbf{Y}(L_2) \\ \vdots \\ \delta \mathbf{Y}(L_n) \end{Bmatrix}$$

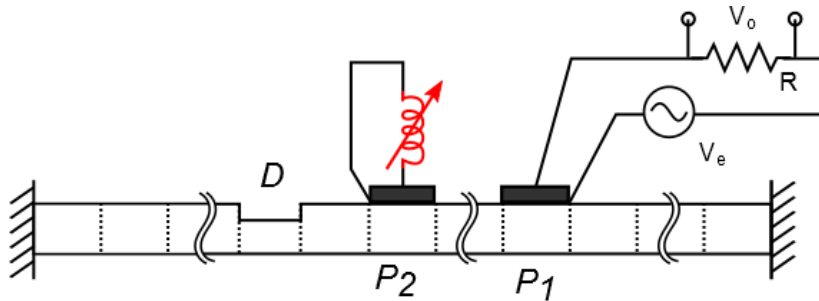
**Significantly  
increased # of  
impedance  
measurements**

***Much more information!***



**Merit of the proposed idea: Original underdetermined inverse problem can be greatly improved!**

# Numerical Analysis: Damage Identification

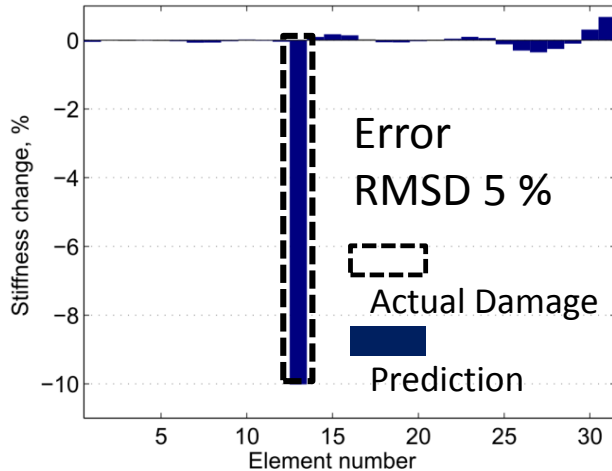


- **Damage:** 10% stiffness loss at the 13<sup>th</sup> element of total 31 elements
- **Data enrichment:** Adaptive piezoelectric circuit with 8 different inductances at the 3<sup>rd</sup> element

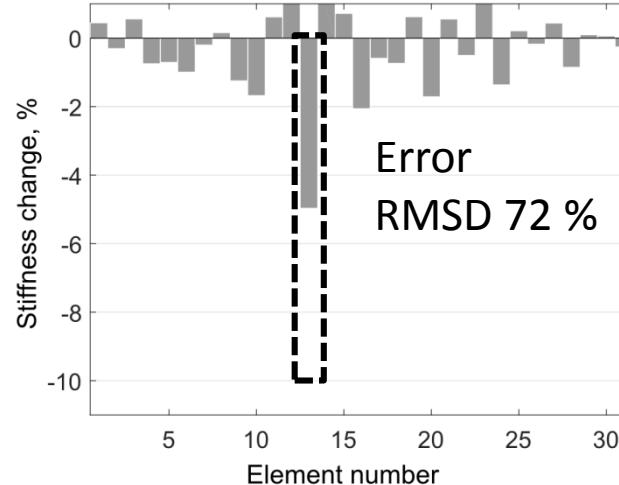
$$RMSD = \sqrt{\frac{\sum(\delta d_{predict} - \delta d_{actual})^2}{\sum(\delta d_{actual})^2}}$$

No.	1	2	3	4	5	6	7	8
L [mH]	134.6	106.4	85.1	68.9	56.4	46.6	38.8	32.6

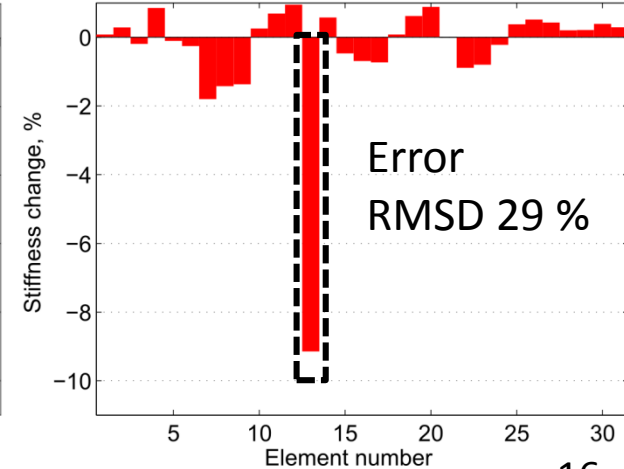
**Conventional method  
wo/ noise**



**Conventional method  
w/ noise, 62dB SNR**

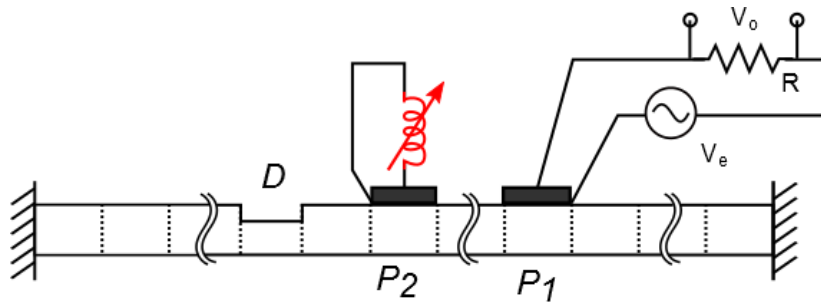


**Data enrichment  
w/ noise, 62dB SNR**





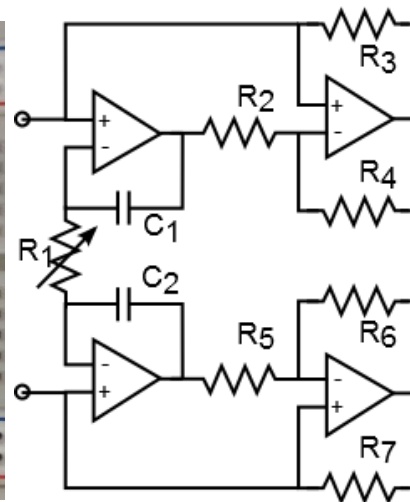
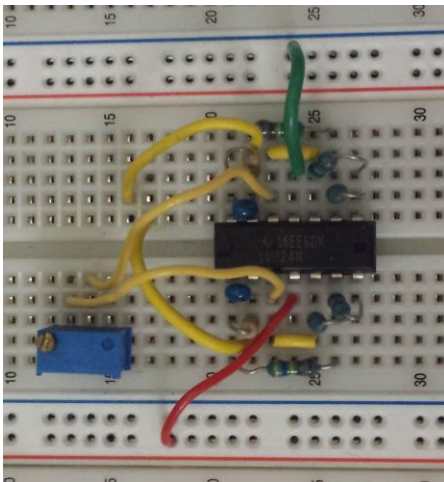
# Experimental Verification



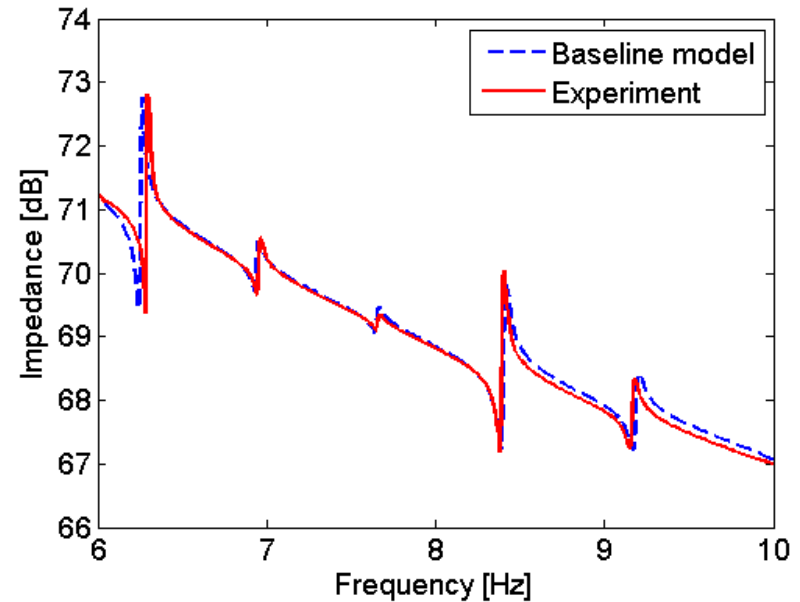
	Elem.	Dimension
<b>Beam structure (Al-2024)</b>	61	627 x 7.21 x 3.175 mm
<b>PZT (PSI-5A4E)</b>	41 / 29	16.8 x 7.09 x 0.191 mm
<b>Damage</b>	25	Notch: 10.4 mm x 90 $\mu$ m → 8.3 % loss

## 7 Inductance tunings in 6 kHz – 10 kHz

No.	1	2	3	4	5	6	7
L, mH	39.5	40.4	41.3	45.3	46.2	63.1	80.1

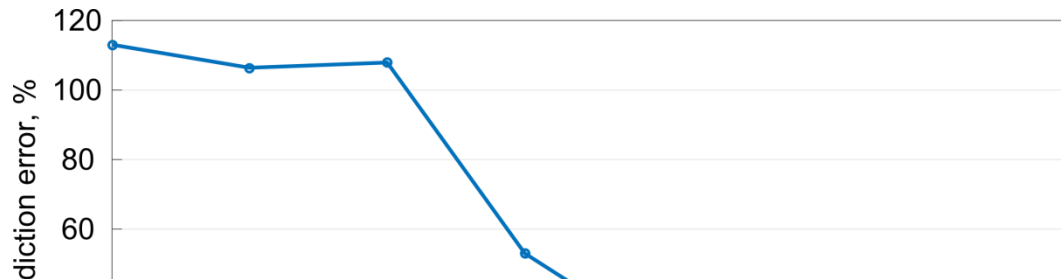
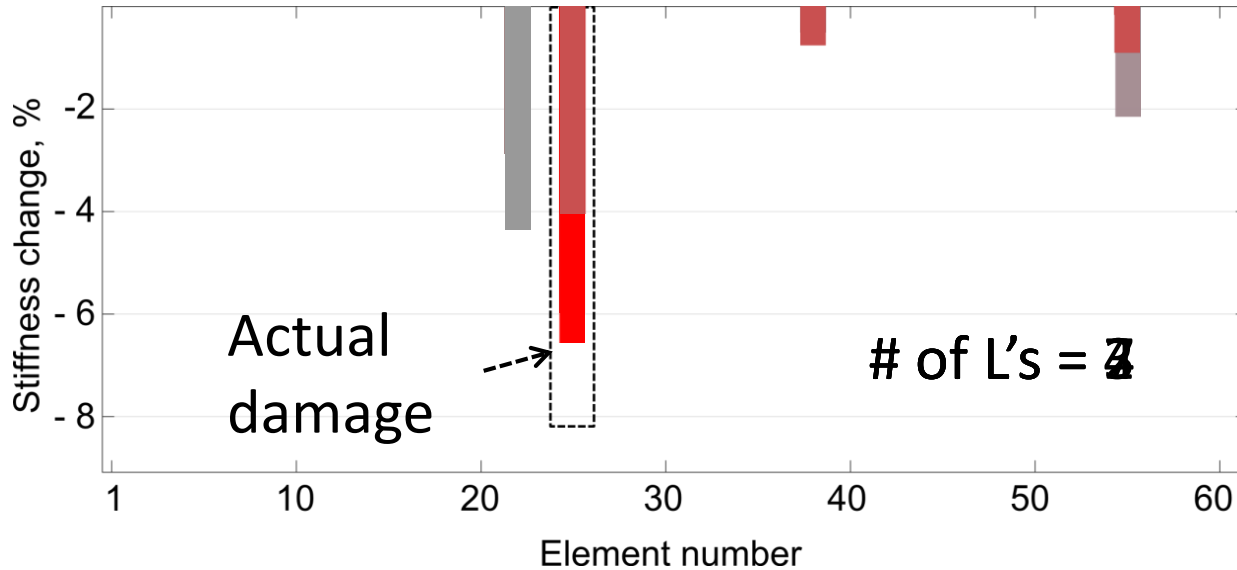


synthetic tunable inductor



# Experimental Verification

- Damage Identification Results



**The concept of data enrichment for improving damage identification is experimentally verified.**

# Problem Statement and New Idea II

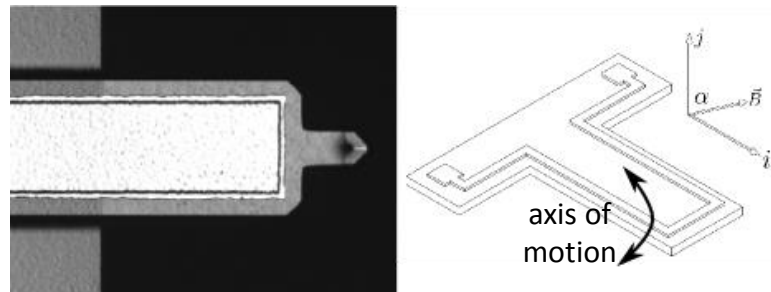
## Problem Statement

- The sensitivity-based inverse problem is significantly underdetermined → Extremely sensitive to small errors such as environmental noise, uncertainty in modeling
- Accurate measurement of damage induced piezoelectric impedance variations, especially with noise

**New Idea: Damage Identification Enhancement with Integrated Bistable and Adaptive piezoelectric Circuitry**

# Bifurcation-Based Sensing in MEMS

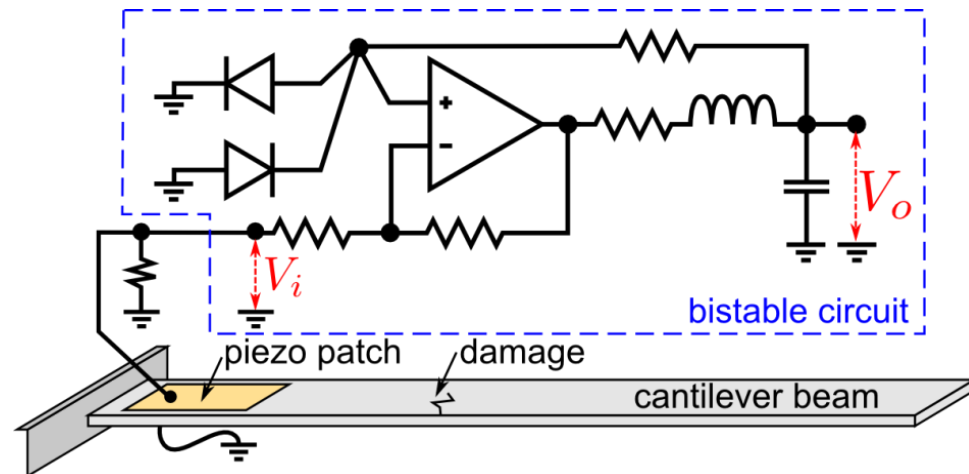
- **Bifurcation:** large qualitative response change (e.g., transition from low- to high-amplitude response, or high-low) due to crossing critical parameters of a nonlinear system
- Microscale (MEMS) mass detection using bifurcations recently studied (Zhang and Turner, 2005; Kumar, et al., 2011)
- Determination of mass accumulation shown to be less susceptible to *noise and damping* than direct frequency peak measurements.



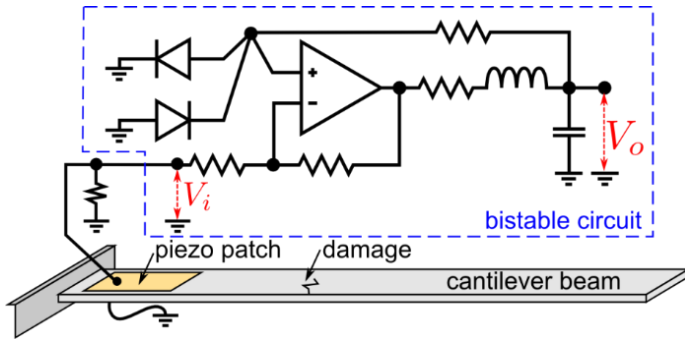
Bifurcation-based mass sensor: piezoelectrically actuated micro-cantilever, Kumar et al 2011

# New Idea: Bistable Circuitry for Bifurcation-Based Sensing for Macro-Structures

- Most structural systems to monitor for damage are not strongly nonlinear → **Need additional means** to introduce strong nonlinearity for bifurcation-based detection
- New Idea: ***Bistable circuitry*** integrated with host structure through piezoelectric transducer
  - The structural response ( $V_i$  from the piezo-transducer) is *input* for the bistable circuit, and can activate the circuit output voltage  $V_o$  bifurcations
  - Negligible back-coupling inhibits interaction between circuit responses and structural dynamics



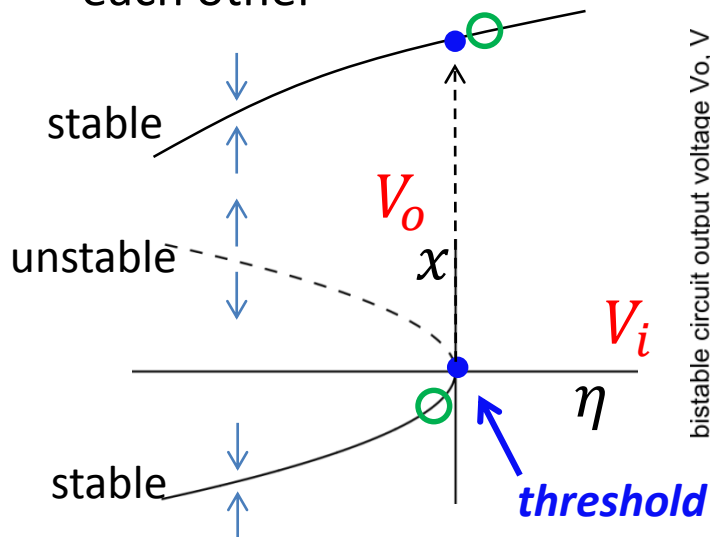
# Bistable Circuitry: Experimental Investigation



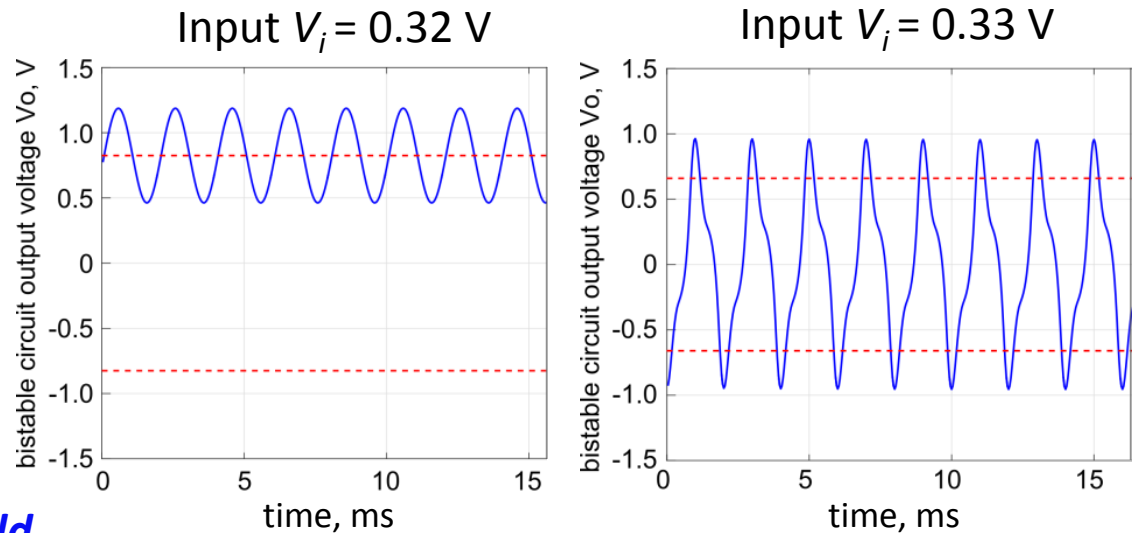
- **Saddle-node bifurcation with respect to input amplitude**
  - Sudden transition from low orbit to high orbit oscillation *when the input amplitude passes a **threshold** → onset of saddle-node bifurcation*

## Saddle-node bifurcation

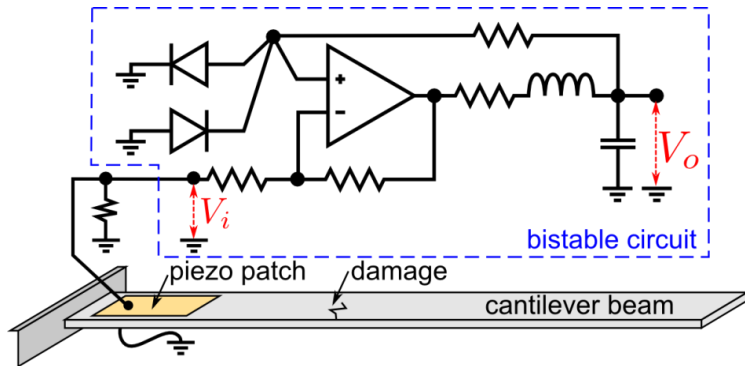
→ two equilibria of a dynamical system *collide* each other



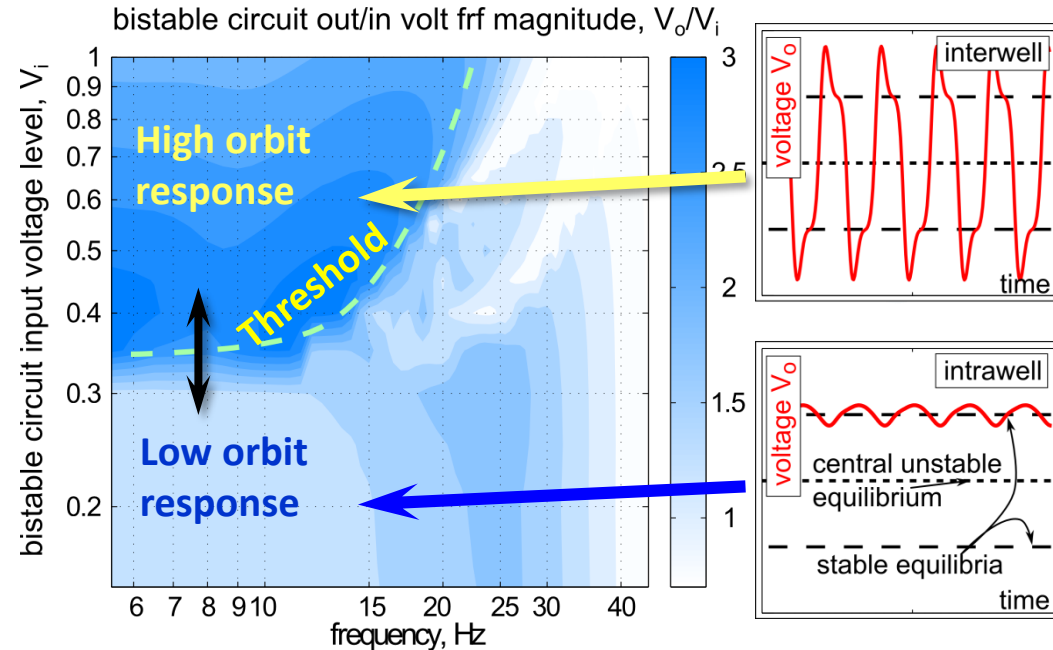
## Bistable circuit response in time



# Bistable Circuitry: Experimental Investigation

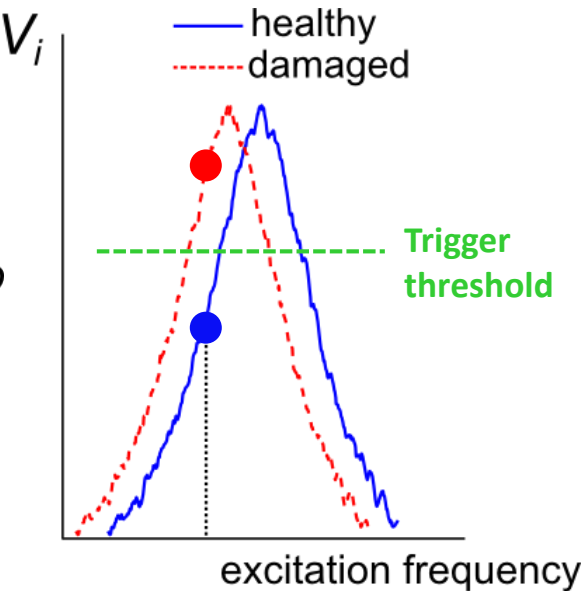
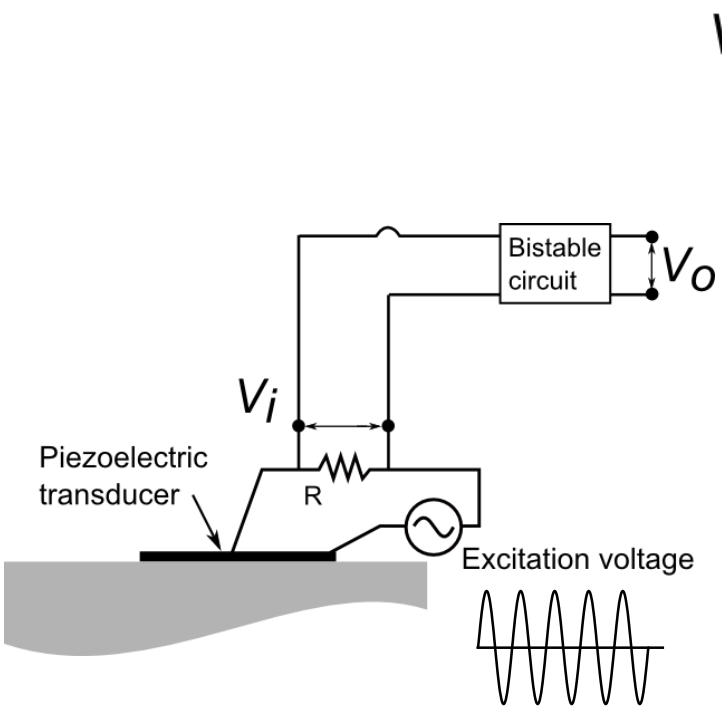


- Bistable circuit response dependent upon excitation frequency and level
- Measured circuit FRF amplitude showing critical region of bifurcation activation with clear threshold

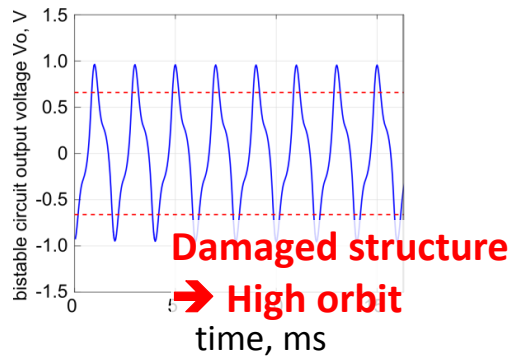
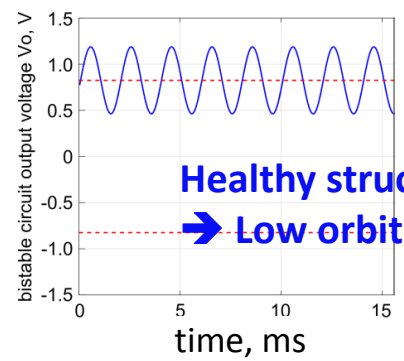


**To detect damage-induced impedance variations -- Tune bistable sensitivity to target a specific structural mode**

# Measuring impedance variations from Bifurcation Activation



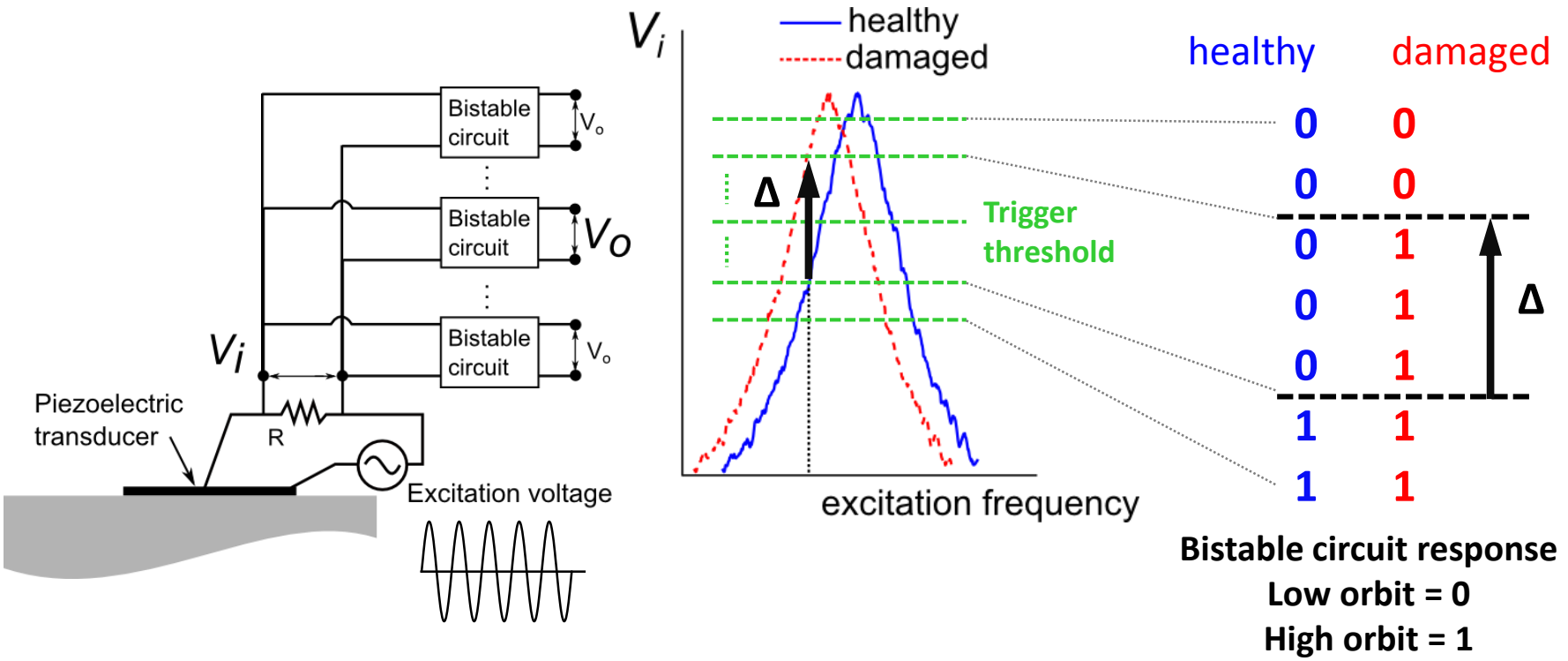
**Bistable circuit response**



- Use host structural response **before/after** damage as input voltage for a bistable circuit
- A critical level of host structural response will trigger bifurcation



# Measuring impedance variations from Bifurcation Activation

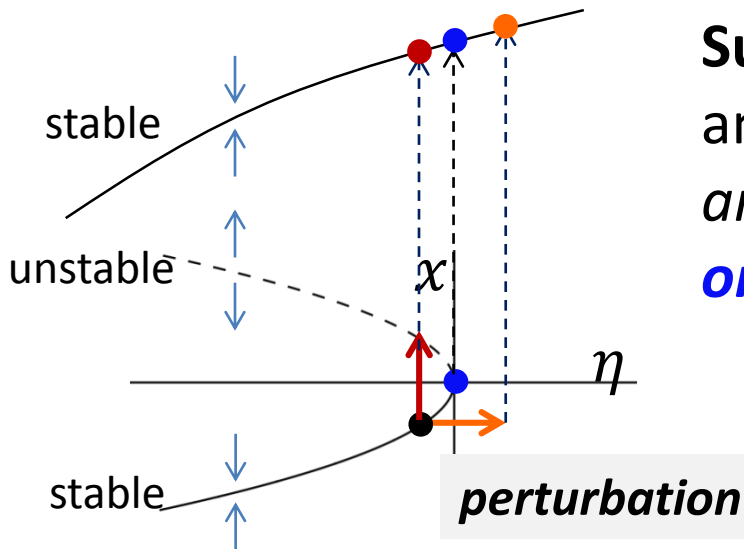


- Use host structural response **before/after** damage as input voltage for *an array of bistable circuit with various threshold levels*
- A critical level of host structural response will trigger bifurcation

**Provide robust measurements of impedance variation for damage ID**

# Non-stationary and stochastic influences on saddle-node bifurcation

- Damage-induced impedance variation measurement *by tracking the onset of bifurcation*



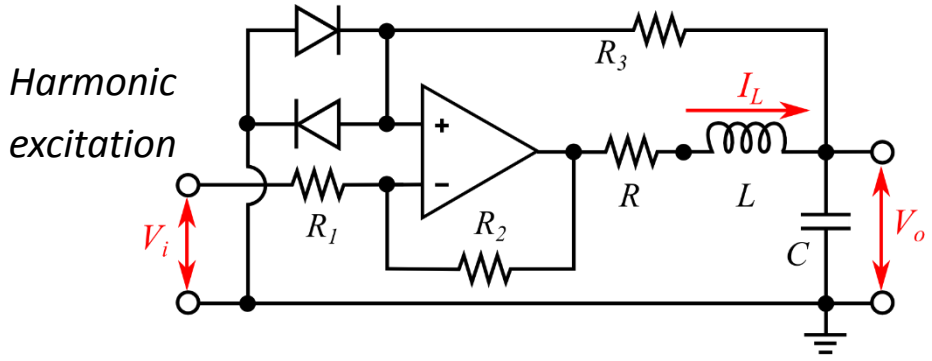
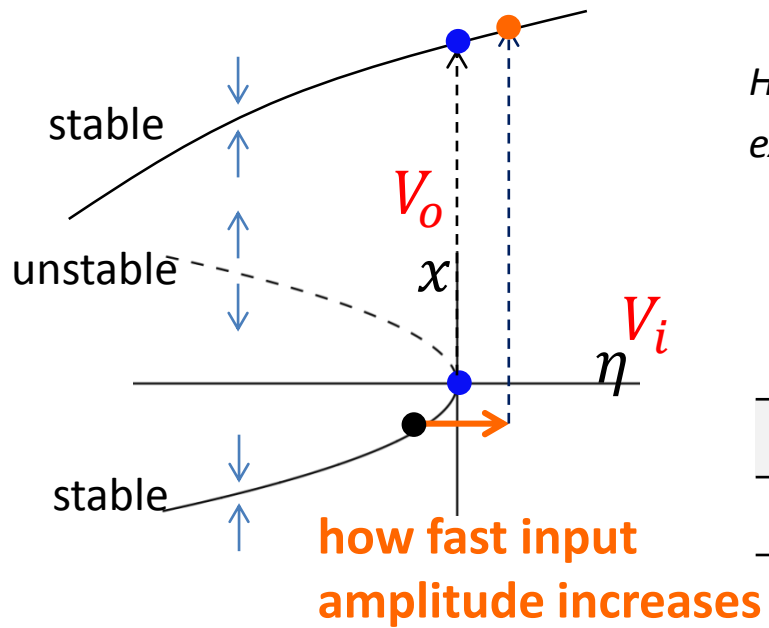
**Sudden transition/escape** to another stable state *when the input amplitude passes a **threshold*** → **onset** of saddle-node bifurcation

**Noise, non-stationary effects influence the onset of bifurcation**

## Saddle-node bifurcation

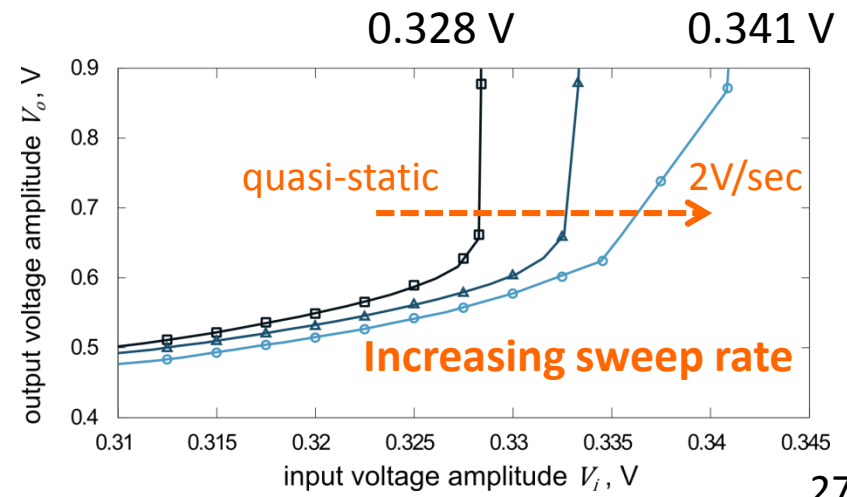
**Accurate assessment on the onset of saddle-node bifurcation is critical for sensing application**

# Non-stationary and stochastic influences on saddle-node bifurcation

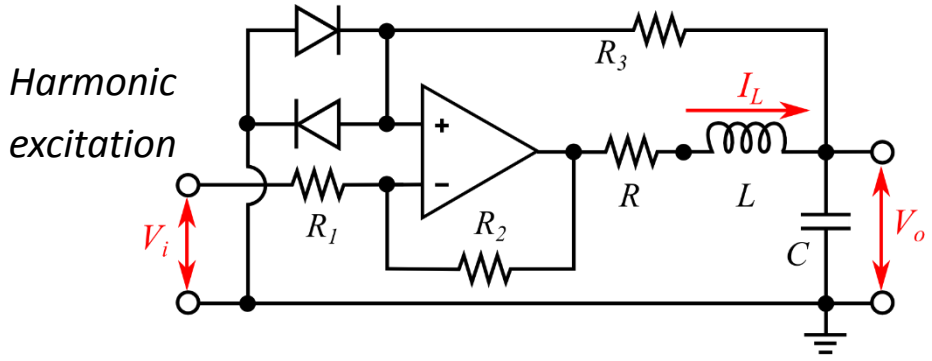
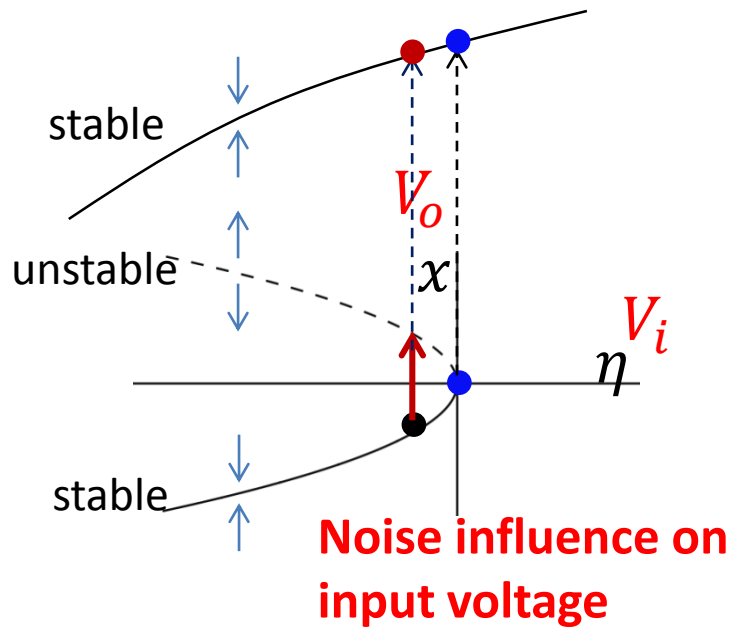


$L, \text{mH}$	$C, \text{nF}$	$R, \Omega$	$R_1, \text{k}\Omega$	$R_2, \text{k}\Omega$	$R_3, \text{k}\Omega$
20	47	32	1	1	2

- **Non-stationary influence**
  - Change input amplitude sweep rate (quasi-static – 2 V/sec)
  - The onset of bifurcation is delayed

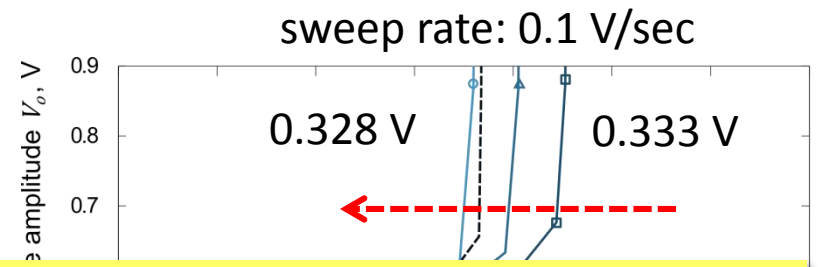


# Non-stationary and stochastic influences on saddle-node bifurcation



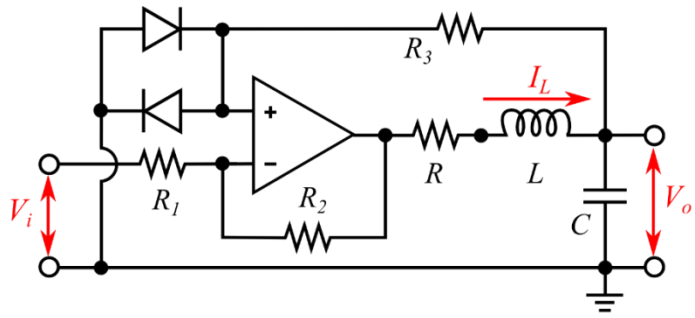
$L, \text{mH}$	$C, \text{nF}$	$R, \Omega$	$R_1, \text{k}\Omega$	$R_2, \text{k}\Omega$	$R_3, \text{k}\Omega$
20	47	32	1	1	2

- Stochastic influence**
  - Additive Gaussian white noise (1 mV – 20 mV)

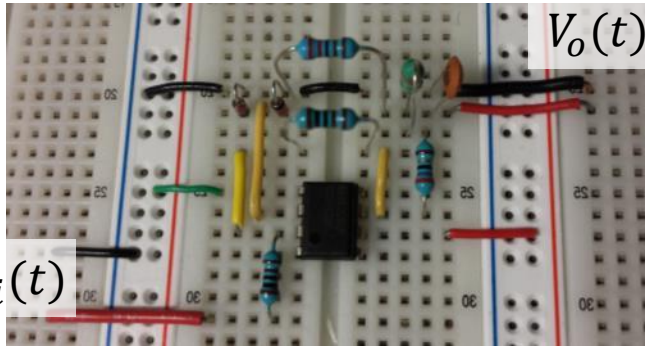
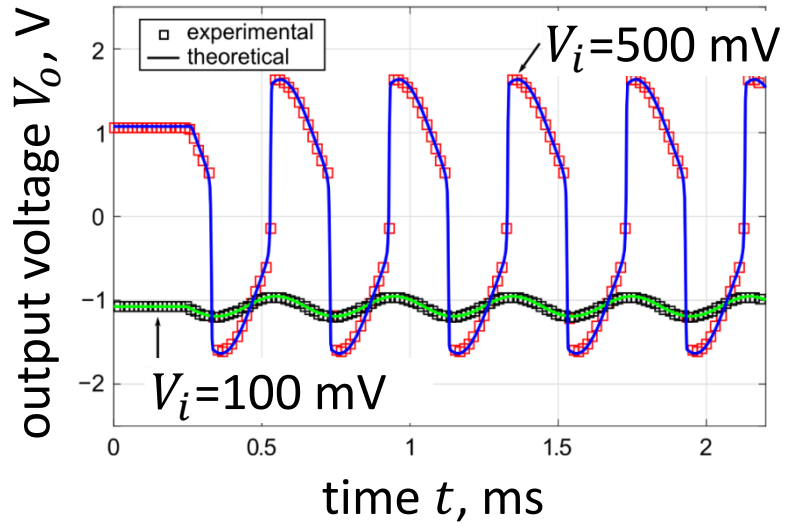


**Saddle-node bifurcation is extremely sensitive to stochastic and non-stationary influences**

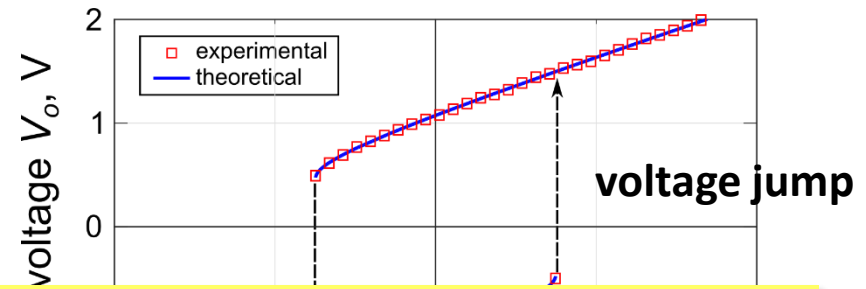
# Theoretical model of the bistable circuit



AC input voltage at 2.5 kHz



DC input voltage sweep



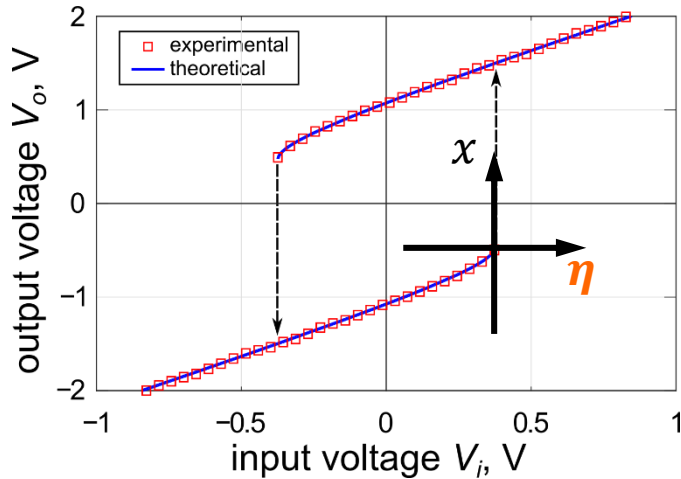
$$L \frac{dI_L}{dt} = (g - 1)V_D - I_L R - 2R_3 I_S \sinh\left(\frac{V_D}{\eta V_T}\right) + (1 - g)V_i$$

**Numerical analysis can be reliably utilized for estimating the experimental results**

$$V_o = V_D + 2R_3 I_S \sinh\left(\frac{V_D}{\eta V_T}\right)$$

input voltage  $V_i$ , V

# Single-parameter stochastic normal form with non-stationary influence



Approximate the local dynamics near bifurcation point as piecewise-linear system

$$RC\dot{V}_o + aV_o + b = V_i + nW$$

Stochastic normal form of non-smooth saddle-node bifurcation

*Gaussian white noise*

$$\dot{x} = \eta + |x| + \varepsilon \xi \quad \langle \xi(t), \xi(\tilde{t}) \rangle = 2\delta(t - \tilde{t})$$

bifurcation parameter

noise strength

Non-stationary influence:  $\eta(t) = \eta_0 + r t$ ,  $\eta_0 < 0$

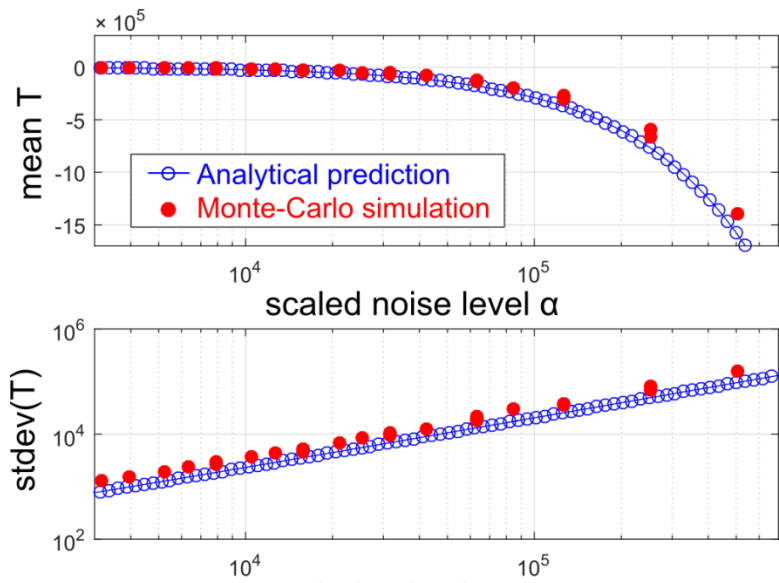
parameter sweep rate

Change of variables  $\tau = \eta r^{-1}$   
 new time scale  $z = x r^{-1}$

Single parameter stochastic normal form with non-stationary influence: Investigate the stochastic and non-stationary influences using **single parameter**, scaled noise  $\alpha$

# Numerical verification of analytical escape probability distribution

- Analytical prediction of escape time  $T$ , where  $z(T) \rightarrow \infty$ 
  - Fokker-Planck equation  $\frac{\partial P(z,t)}{\partial t} = -\frac{\partial}{\partial z} [(t + z^2)P] + \alpha^2 \frac{\partial^2 P}{\partial z^2}$
  - Kramer's escape rate  $P(T) = W(T) \exp\left(-\int_{t_0}^T W(t)dt\right) \quad W(t) = \frac{\sqrt{-t}}{\pi} \exp\left(-\frac{4(-t)^{3/2}}{3\alpha^2}\right)$
- Numerical verification
  - Monte-Carlo method: solve stochastic differential equation of the bistable circuit via Euler-Maruyama approach

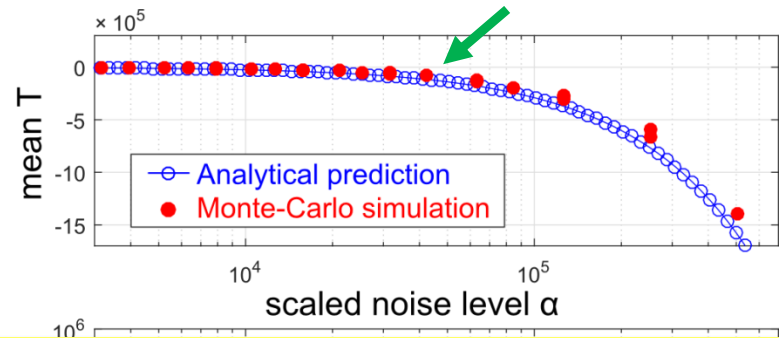


- Noise level  
 → 2.5, 5, 10, 20 40 mV rms
- Input amplitude sweep rate  
 → 10, 20, 40, 60, 80, 200 mV/sec

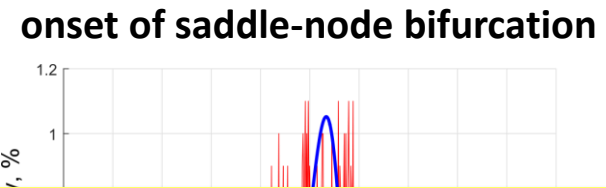
fast sweep/  
weak noise ←-----→ Strong noise/  
slow sweep

# Numerical verification of analytical escape probability distribution

- Analytical prediction of escape time  $T$ , where  $z(T) \rightarrow \infty$ 
  - Fokker-Planck equation  $\frac{\partial P(z,t)}{\partial t} = -\frac{\partial}{\partial z} [(t + z^2)P] + \alpha^2 \frac{\partial^2 P}{\partial z^2}$
  - Kramer's escape rate  $P(T) = W(T) \exp\left(-\int_{t_0}^T W(t)dt\right)$   $W(t) = \frac{\sqrt{-t}}{\pi} \exp\left(-\frac{4(-t)^{3/2}}{3\alpha^2}\right)$
- Numerical verification
  - Monte-Carlo method: solve stochastic differential equation of the bistable circuit via Euler-Maruyama approach



Noise level: 10 mV rms  
Sweep rate: 40 mV/sec



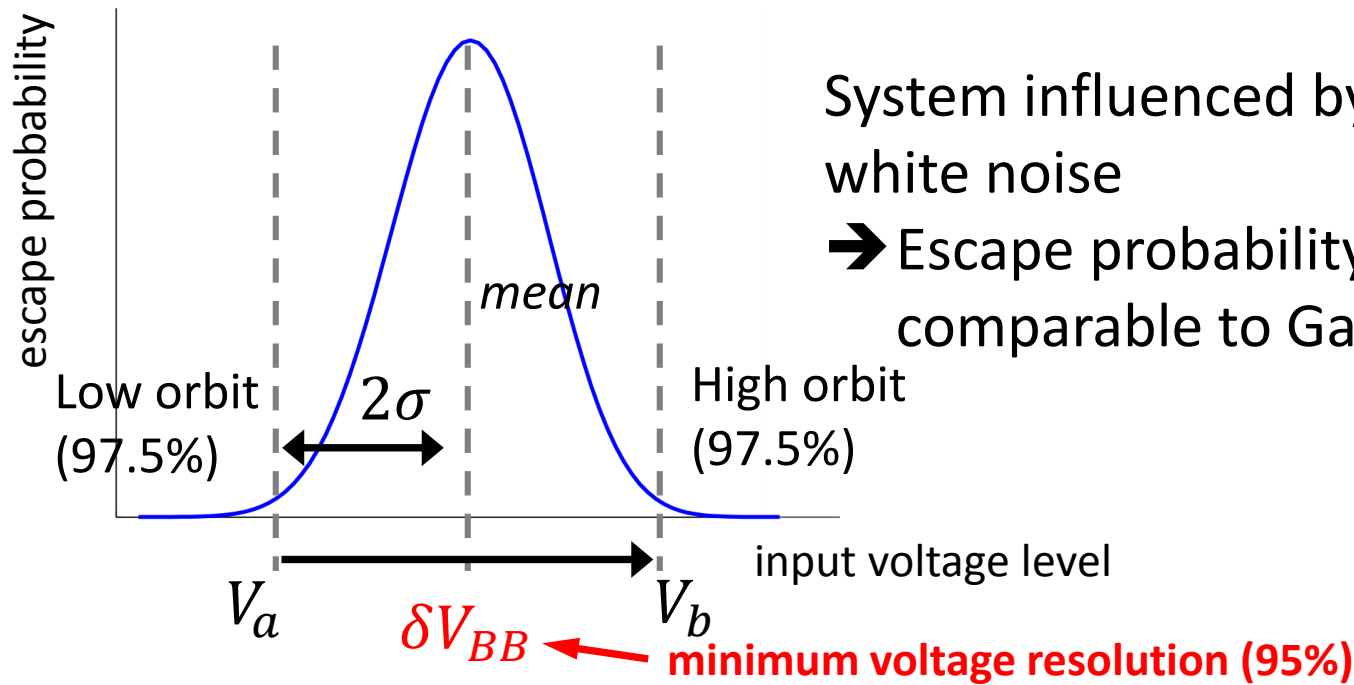
**Analytical prediction on the onset of saddle-node bifurcation is verified for various non-stationary and stochastic conditions**

weak noise ← → slow sweep

escape voltage, mV



# Escape probability distribution and bifurcation-based sensing resolution



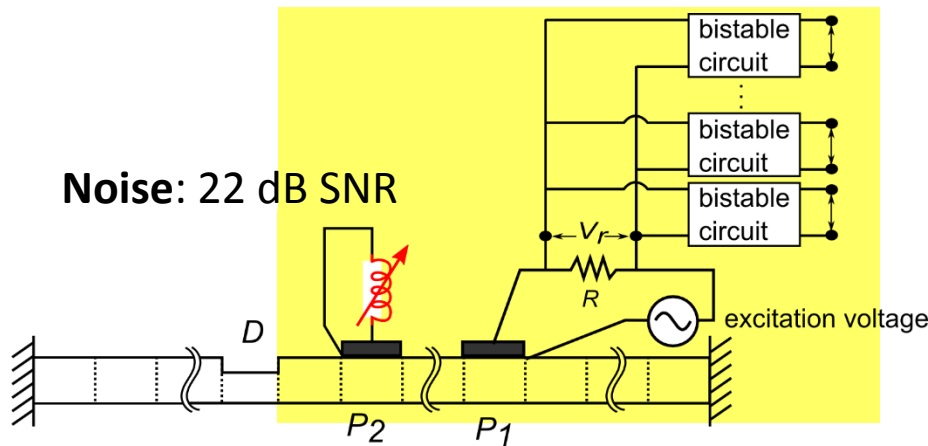
System influenced by Gaussian white noise  
 → Escape probability distribution comparable to Gaussian.

- Case: noise level (10 mV), sweep rate (40 mV/s)

**Theoretical framework enables to determine the enhanced minimum resolution of bifurcation-based sensing approach**

- A

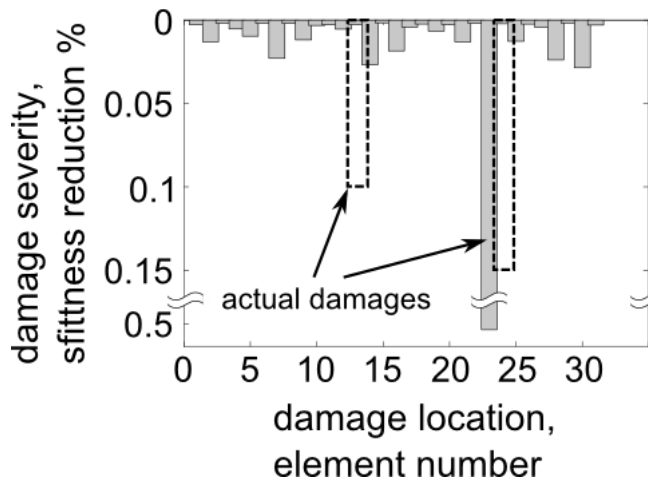
# Damage Identification with Integrated Bistable and Adaptive Piezoelectric Circuitry



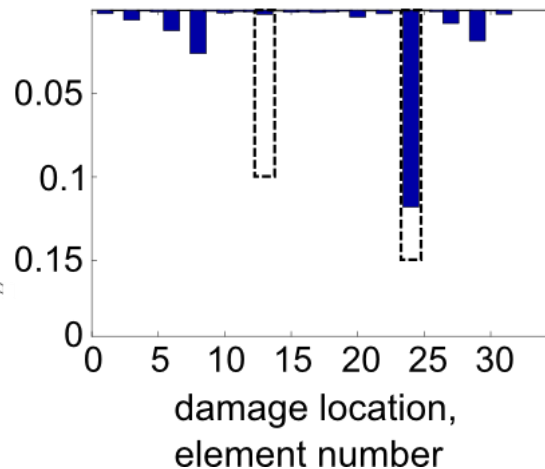
- **Data enrichment**
  - Adaptive piezoelectric circuit with 7 different inductances
- **Impedance measurement**
  - Bistable circuitry integrated with piezoelectric circuitry

**Damages:** 0.1%, 0.15% element stiffness reduction @ 13<sup>th</sup>, 24<sup>th</sup> element

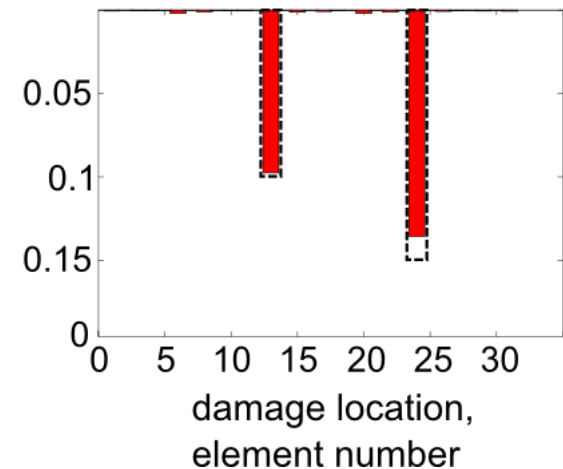
with no data enrichment



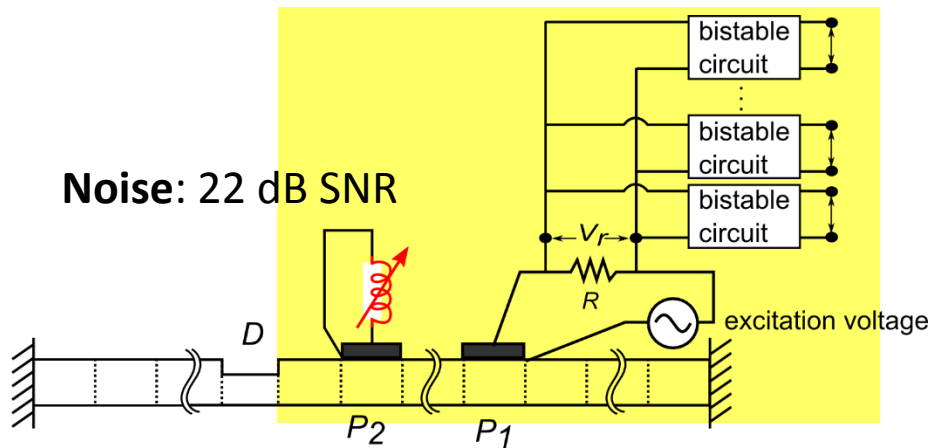
with data enrichment



bifurcation-based measurement with data enrichment

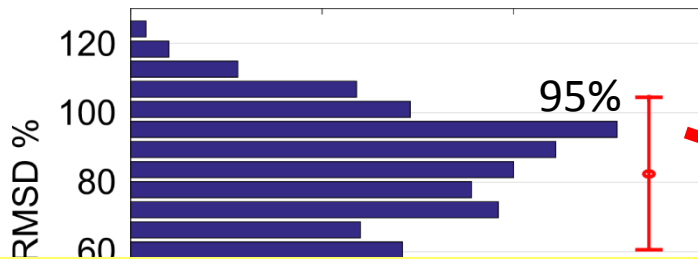


# Damage Identification with Integrated Bistable and Adaptive Piezoelectric Circuitry

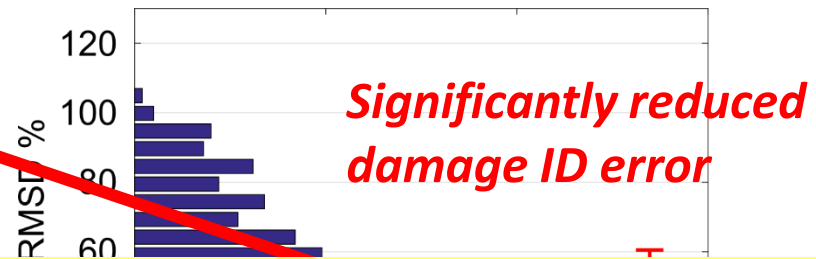


- Damage identification for various cases: 1000 combinations of
  - 0.5, 1, 1.5, or 2% element stiffness reduction,
  - 1, 2, or 3 locations of damages.

Data enrichment with **conventional** measurement



Data enrichment with **bifurcation-based** measurement



*Significantly reduced damage ID error*

**Damage identification via integrated bistable and adaptive piezoelectric circuitry is much more accurate and robust**

# Summary and Conclusion

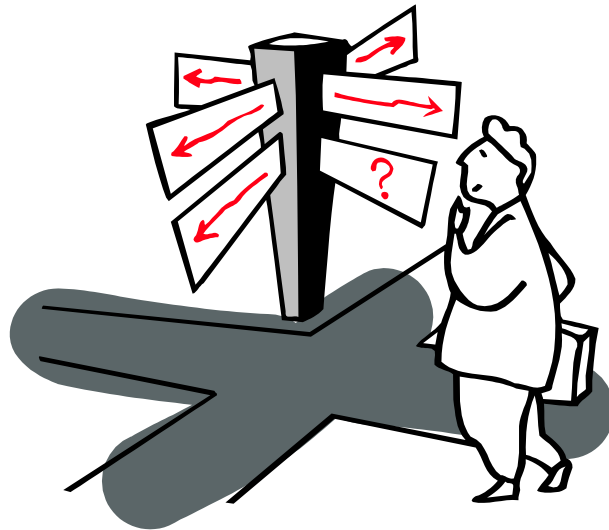
- **Develop integrated bistable and adaptive piezoelectric circuitry**
  - ➔ Fundamental improvement of underdetermined inverse problem for damage identification
- **Create bistable circuitry sensing platform**
  - ➔ Key element to extend the applicability of bifurcation-based sensing scheme
- **Establish novel analysis on stochastic and dynamic saddle-node bifurcation**
  - ➔ Simple and accurate prediction of the critical conditions of various disciplines that exhibit saddle-node bifurcation
  - ➔ Fundamental understanding of the sensing limit under noise and non-stationary influences.

# Future Plans

## Vision: Advance System Monitoring and Sensing Strategies for Sustainable and Resilient System Health Management

- **Structural health monitoring**
  - ➔ Measurement and modeling uncertainty quantification/management
  - ➔ Long-term goal: Prognosis and decision-making for maintenance
  - ➔ Collaboration with UTC in engine health monitoring
- **Bifurcation-based bistable circuitry sensors**
  - ➔ Various bistable circuitry architecture and optimal parameter design
  - ➔ Application with MEMS sensors for HVAC systems → medical, safety
- **Forecasting critical transitions in complex systems**
  - ➔ Nonlinear bifurcation prediction + model-less data-driven approach
  - ➔ Ecological and climate systems to aero-elasticity in aircraft, power grid systems

# The End



Questions?