Managing systemic risk

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Networks are important!

"Ford went to Capitol Hill in late 2008 pushing for the rescue of its rivals, GM and Chrysler ... GM received $49.5 billion,... Chrysler Group received $10.5 billion in bailout funds"

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"Without financing during bankruptcy, GM and Chrysler would have had to go out of business, taking down many suppliers. That would have likely caused bankruptcies at the healthier automakers such Ford Motor, who would not have been able to get the parts they needed to build cars."

– CNN

Systemic Risk

'system' $\equiv$ collection of 'entities'.

Examples:

- firms in an economy
- business units in a company
- suppliers, sub-contractors, etc. in a supply chain network
- generating stations, transmission facilities, etc. in a power network

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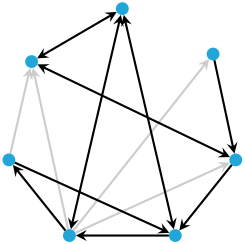
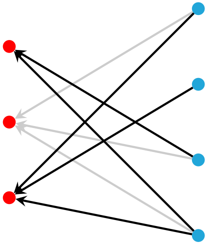
Systemic risk refers to the risk of the entire system. Involves:

- the simultaneous analysis of outcomes across all entities in a system
- the possibility of complex interactions across the network

Bank Lending vs Asset-Firm Holding



Cross-firm Lending



Asset-firm Holding

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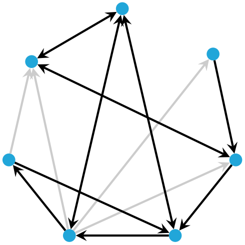
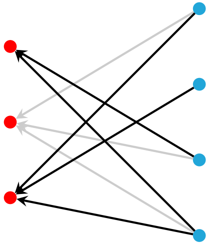


Cross-firm Lending



Asset-firm Holding

WSJ OpEd, Peter Wallison, 10 February, 2012

"None of these firms was weakened by its exposure to Lehman or anyone else. They were weakened by the fact that virtually all of them held – or were suspected of holding – large amounts of what the media came to call toxic assets."

Three different approaches

Risk management

- Portfolios known: distributions known but realizations unknown
- Goal: Apportion total risk to various entities

Stylized dynamic models

- Portfolio update rules known
- Goal: Understand the characteristics of resilient networks

Feedback analysis

- Signed directed graphs (SDG) to model feedback
- Goal: Analysis of particular networks

Risk management

$\mathcal{F} \equiv$ set of nodes in the network (firms, suppliers, edges in a graph)

$\tilde{X}_i =$ random loss of node i $\qquad$ $\tilde{X}_{\mathcal{F}} =$ random loss of the network

Risk management

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$\tilde{X}_i =$ random loss of node i $\qquad$ $\tilde{X}_{\mathcal{F}} =$ random loss of the network

Goal: Measure for the "acceptability" of $\tilde{X}_{\mathcal{F}}$

- risk measure $\rho(\cdot)$: $\rho(\tilde{X}_{\mathcal{F}})$ is the "risk" of $\tilde{X}_{\mathcal{F}}$
- Allocate $\rho(\tilde{X}_{\mathcal{F}})$ to individual entities i
- Incentive compatibility

Examples of Financial Systemic Risk Measures

- $\mathcal{F}$ = firms in the economy
- $X_{i,\omega}$ = loss of a firm i in scenario ω

Example. (Systemic Expected Shortfall)

$$\mathsf{CVaR}_\alpha\left(\left\{\sum_{i\in\mathcal{F}} X_{i,\omega}\right\}\right)$$

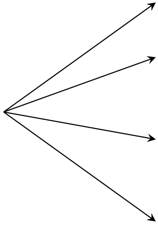
[Acharya et al., 2010; Brownlees, Engle 2010]

Example. (Deposit Insurance)

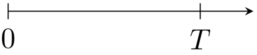
$$\mathbb{E}^*\left[\sum_{i\in\mathcal{F}} X_{i,\omega}^+\right]$$

[e.g., Lehar, 2005; Huang et al., 2009]

Systemic Risk Measures



Scenario	Firm 1	Firm 2		Firm $|\mathcal{F}|$
ω_1	X_{1,ω_1}	X_{2,ω_1}	$\cdots$	$X_{|\mathcal{F}|,\omega_1}$
ω_2	X_{1,ω_2}	X_{2,ω_2}	$\cdots$	$X_{|\mathcal{F}|,\omega_2}$,
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$\omega_{|\Omega|}$	$X_{1,\omega_{|\Omega|}}$	$X_{2,\omega_{|\Omega|}}$	$\cdots$	$X_{|\mathcal{F}|,\omega_{|\Omega|}}$



$\Omega =$ set of scenarios

$\mathcal{F} =$ set of firms (entities in the system)

$X \in \mathbb{R}^{\Omega \times \mathcal{F}}$

$X_{i,\omega} =$ loss for firm i in scenario ω

Example

- 3 firms in 3 future scenarios (equally likely)
- Loss matrix (+ Loss; - Profit)

Scenario	Firm 1	Firm 2	Firm 3
ω_1	+50	-40	+20
ω_2	-40	+50	-40
ω_3	+20	+20	+50

Questions:

- What is the total "risk" of the economy?
- How does one "allocate" this risk to each of the three firms?

Systemic Risk Measures: Definition

- $\Omega =$ set of scenarios $\quad \mathcal{F} =$ set of firms (entities in the system)
- $X_{i,\omega} =$ loss for firm i in scenario $\omega \quad X \in \mathbb{R}^{\Omega \times \mathcal{F}}$
- $X_\omega =$ loss vector in scenario ω

Systemic Risk Measures: Definition

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Definition. A systemic risk measure $\rho\colon \mathbb{R}^{\Omega \times \mathcal{F}} \to \mathbb{R}$ satisfies:

(i) *Monotonicity:* if $X \geq Y$, then

$$\rho(X) \geq \rho(Y)$$

(ii) *Positive homogeneity*: for all $\alpha \geq 0$,

$$\rho(\alpha X) = \alpha \rho(X)$$

(iii) *Normalization:* $\rho(\mathbf{1}_{\mathcal{E}}) = |\mathcal{F}|$

Systemic Risk Measures: Definition

Given $x, y \in \mathbb{R}^{\mathcal{F}}$, define the ordering $x \succeq_\rho y$

$$x \succeq_\rho y \quad \Longleftrightarrow \quad \rho\left(\begin{bmatrix} x^\top \\ x^\top \\ \vdots \\ x^\top \end{bmatrix}\right) \geq \rho\left(\begin{bmatrix} y^\top \\ y^\top \\ \vdots \\ y^\top \end{bmatrix}\right)$$

i.e. an economy with outcome x (resp. y) in all states Ω

Definition. (con't.)

(iv) *Preference consistency*: if $X_\omega \succeq_\rho Y_\omega$ for all scenarios ω, then

$$\rho(X) \geq \rho(Y)$$

Systemic Risk Measures: Definition

Definition. (con't.)

(v) *Convexity*: for all $0 \leq \alpha \leq 1$, $\bar{\alpha} = 1 - \alpha$

(a) *Outcome convexity:* if

$$Z = \alpha X + \bar{\alpha} Y \tag{1}$$

then, $\rho\big(Z\big) \leq \alpha\rho(X) + \bar{\alpha}\rho(Y)$

(b) *Risk convexity:* if for all scenarios $\omega \in \Omega$,

$$\rho(Z_\omega, \ldots, Z_\omega) = \alpha\rho(X_\omega, \ldots, X_\omega) + \bar{\alpha}\rho(Y_\omega, \ldots, Y_\omega), \tag{2}$$

then, $\rho\big(Z\big) \leq \alpha\rho(X) + \bar{\alpha}\rho(Y)$

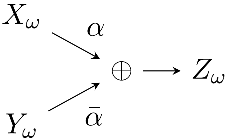
Two different notions of **diversity**

- One allows cross-subsidization
- Other removes randomness

Systemic Risk Measures: Definition

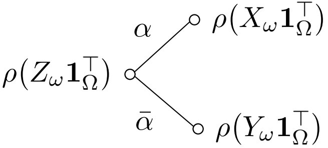
Definition. (con't.)

1. Outcome convexity: Increasing diversity reduces risk



$$\Rightarrow \quad \rho(Z) \leq \alpha\rho(X) + \bar{\alpha}\rho(Y)$$

2. Risk convexity: Removing randomness reduces risk



$$\Rightarrow \quad \rho(Z) \leq \alpha\rho(X) + \bar{\alpha}\rho(Y)$$

Structural Decomposition

Definition. An aggregation function is a function $\Lambda\colon \mathbb{R}^{\mathcal{F}} \to \mathbb{R}$ that is monotonic, positively homogeneous, convex, and normalized so that $\Lambda(\mathbf{1}_{\mathcal{F}}) = |\mathcal{F}|$.

Aggregation function: aggregates risk across firms in a given scenario

Structural Decomposition

Definition. An aggregation function is a function $\Lambda\colon \mathbb{R}^{\mathcal{F}} \to \mathbb{R}$ that is monotonic, positively homogeneous, convex, and normalized so that $\Lambda(\mathbf{1}_{\mathcal{F}}) = |\mathcal{F}|$.

Aggregation function: aggregates risk across firms in a given scenario

Theorem. A function $\rho\colon \mathbb{R}^{\Omega\times\mathcal{F}} \to \mathbb{R}$ is a systemic risk measure with $\rho(\mathbb{R}^{|\Omega|\times|\mathcal{F}|}) = \mathbb{R}$ if, and only if, there exists

- an aggregation function Λ
- coherent single-firm base risk measure ρ_0 such that

$$\rho(X) = (\rho_0 \circ \Lambda)(X) \triangleq \rho_0\left(\Lambda(X_1), \Lambda(X_2), \ldots, \Lambda(X_{|\Omega|})\right)$$

Example: Economic Systemic Risk Measures

- $\mathcal{F}$ = firms in the economy
- $X_{i,\omega}$ = loss of a firm i in scenario ω

Example. (Systemic Expected Shortfall)

$$\Lambda_{\mathsf{total}}(x) \triangleq \sum_{i \in \mathcal{F}} x_i, \qquad \rho_{\mathsf{SES}}(X) \triangleq (\mathsf{CVaR}_\alpha \circ \Lambda_{\mathsf{total}})(X)$$

[Acharya et al., 2010; Brownlees, Engle 2010]

Example. (Deposit Insurance)

$$\Lambda_{\mathsf{loss}}(x) \triangleq \sum_{i \in \mathcal{F}} x_i^+, \qquad \rho_{\mathsf{DI}}(X) \triangleq \mathbb{E}^* \left[\Lambda_{\mathsf{loss}}(X_\omega)\right] = \mathbb{E}^* \left[\sum_{i \in \mathcal{F}} X_{i,\omega}^+\right]$$

[e.g., Lehar, 2005; Huang et al., 2009]

Example: Resource Allocation

- $\mathcal{A} =$ a set of activities
- $\mathcal{F} =$ a set of capacitated resources
- $X_{i,\omega} =$ shortage of resource i in scenario ω

Aggregation function:

$$\Lambda_{\mathsf{RA}}(x) \triangleq \begin{array}{ll} \min\limits_{u} & \sum\limits_{a \in \mathcal{A}} c_a u_a \\ \text{subject to} & \sum\limits_{a \in \mathcal{A}} b_{ia} u_a \geq x_i, \quad \forall\, i \in \mathcal{F} \\ & u \in \mathbb{R}^{\mathcal{A}} \end{array}$$

where

- $u_a =$ reduction in level of activity a (decision variable)
- $c_a =$ per-unit cost of reductions in activity a
- $b_{ia} =$ per-unit consumption of resource i by activity a

Example: Eisenberg-Noe Contagion Model

- $\mathcal{F}$ = firms, who have assets and obligations to each other
- Π_{ij} = fraction of the debt of firm i owed to firm j
- x = losses on the asset portfolio of firms

Aggregation Function: $\gamma > 1$

$$\begin{aligned} \Lambda_{\mathsf{CM}}(x) \triangleq \quad & \min_{y \in \mathbb{R}_+^{\mathcal{F}},\ b \in \mathbb{R}_+^{\mathcal{F}}} && \sum_{i \in \mathcal{F}} y_i + \gamma \sum_{i \in \mathcal{F}} b_i \\ & \text{subject to} && b_i + y_i \geq x_i + \sum_{j \in \mathcal{F}} \Pi_{ji} y_j, \quad \forall\ i \in \mathcal{F}. \end{aligned}$$

where loss x_i in firm i is covered by

- reducing payments by y_i
- borrowing b_i from the regulator

Risk attribution

Dual representation for risk $\rho(X)$:

$$\rho(X) = \begin{array}{ll} \underset{\bar{\pi},\Xi}{\text{maximize}} & \sum_{i\in\mathcal{F}} \sum_{\omega\in\Omega} \Xi_{i,\omega} X_{i,\omega} \\ \text{subject to} & (1,\bar{\pi}) \in \mathcal{A}^* \\ & (\bar{\pi}_\omega, \Xi_\omega) \in \mathcal{B}^*, \ \forall\, \omega \in \Omega \\ & \bar{\pi} \in \mathbb{R}^\Omega, \ \Xi \in \mathbb{R}^{\mathcal{F}\times\Omega} \end{array}$$

Risk attributed of firm i: $y_i^* = \sum_{\omega\in\Omega} \Xi^*_{i,\omega} X_{i,\omega}$

Theorem. (No Undercut) Given $\alpha \in \mathbb{R}_+^{\mathcal{F}}$, define

$$r(\alpha) \triangleq \rho\big(\alpha_1 x_1; \ldots; \alpha_{|\mathcal{F}|} x_{|\mathcal{F}|}\big)$$

Then,

$$\alpha^\top y^* \leq r(\alpha)$$

Generalization of attribution scheme of Aumann & Shapley (1974), Denault (2001), Buch & Dorfleitner (2008).

Example: Risk Attribution

- 3 firms in 3 future scenarios (equally likely)
- $\rho_{\mathsf{SES}}(X) \triangleq (\mathsf{CVaR}_{1/3} \circ \Lambda_{\mathsf{total}})(X) = \mathsf{CVaR}_{1/3}(x_1 + x_2 + x_3) = 30$

Scenario	Firm 1	Firm 2	Firm 3
ω_1	+50	-40	+20
ω_2	-40	+50	-40
ω_3	+20	+20	+50
Risk Attribution	20/3	20/3	50/3

Structural decomposition extends broadly

Homogeneous Systemic Risk Measures:

- monotone, +vely homogeneous, preference consistent, **not** convex
- structural decomposition exists
  Homogeneous single-firm base risk measure
  Homogeneous aggregation function

Convex Systemic Risk Measures:

- monotone, convex, preference consistent, **not** +vely homogeneous
- structural decomposition exists
  convex single-firm base risk measure
  convex aggregation function

Key idea: Preference consistency allows for the structural decomposition

Further extensions

General probability spaces: Kromer, Overbeck and Zilch. *Systemic risk measures on general probability spaces*. 2014.

- Coherent systemic risk measures
- Convex systemic risk measures
- Monotone positively homogeneous systemic risk measures

More general risk measures: Biagini, Fouque, Frittelli Meyer-Brandis. *A unified approach to systemic risk measures via acceptance sets*. 2015.

- $\rho(X) = \text{inf}\{\pi(Y) : \Lambda(X - Y) \in \mathcal{A}\}$, Y = capital injection

Set-valued measures of systemic risk Feinstein, Rudloff, Weber. *Measures of Systemic Risk*. 2015.

- $\rho(X) = \{Y : \Lambda(X + Y) \in \mathcal{A}^Y\} \subseteq \mathbb{R}^{|\mathcal{F}|}$

Asset-firm networks

$X_{\mathcal{F}}$ random losses of $\mathcal{F}$ nodes in Ω scenarios

- $X_{\mathcal{F}}$ is completely exogenous
- The nodes do not take any actions

The network consists of just the firms. Only captures cross-firm lending

- defaults, hair-cut, funding liquidity

Firms also interact via commonly held assets

- MBS, fire sales, volatility, risk-aversion

Model: Assets, firms and portfolio rules

$\mathcal{A}$ = set of assets. $\mathcal{F}$ = set of firms.

- Portfolio rules for firms: $\Pi \in \mathbb{R}^{|\mathcal{A}| \times |\mathcal{F}|}$

$$\Pi_{i,h} = \text{fraction of wealth of firm } h \text{ invested in asset } i$$

- Portfolio rules $\Pi(q, x)$ rules depend on ...
  - prices q
  - exogenous (risk) factors x
  - could also depend on wealth w (in the paper!)

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Example: CRRA utility with risk aversion β_h, log-normal payoffs $\log(p_h) \sim N(\mu_h, \Sigma_h)$.

$$\Pi_h(q, \mu_h, \Sigma_h, r_f) = \frac{1}{\beta_h} \Sigma_h^{-1} \left(\mu_h - \log(q) - r_f \mathbf{1} + \frac{1}{2} \mathrm{diag}\{\Sigma_h\} \right)$$

Prices are endogenous!

Market clearing implies

$$q = D^{-1}\Pi(q,x)w = D^{-1}\Pi(q,x)(\theta^0(x) + \Theta^T q)$$

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Implicit function theorem

$$\frac{\partial q}{\partial x} = D^{-1} \quad \left[I - \Pi\bar{\Theta}^T - HD^{-1}\right]^{-1} \quad \left[\frac{\partial \Pi}{\partial x} w + \Pi \frac{\partial \theta^0}{\partial x}\right]$$

Network Effect   Direct Effect

$$\Delta q = D^{-1} \quad \left[I - \Pi\bar{\Theta}^T - HD^{-1}\right]^{-1} \quad \left[\frac{\partial \Pi}{\partial x} w + \Pi \frac{\partial \theta^0}{\partial x}\right] \quad \Delta x$$

Direct Effect, propagated via Network Effect, forms price change.

Network Effect

$$\frac{\partial q}{\partial x} = D^{-1}\left[I - \Pi\bar{\Theta}^T - HD^{-1}\right]^{-1}\left[\frac{\partial \Pi}{\partial x} w + \Pi \frac{\partial \theta^0}{\partial x}\right]$$

Two components

- $\Pi\bar{\Theta}^T \in \mathbb{R}^{|\mathcal{A}|\times|\mathcal{A}|}$: holding-induced cross-asset interaction

$$(\Pi\bar{\Theta}^T)_{ij} = \sum_{h=1}^{|\mathcal{F}|} \Pi_{ih}\bar{\Theta}_{jh}.$$

- $H \in \mathbb{R}^{|\mathcal{A}|\times|\mathcal{A}|}$: wealth-weighted cross-asset portfolio sensitivity

$$H_{ij} = \sum_{h=1}^{|\mathcal{F}|} \frac{\partial \Pi_{ih}}{\partial q_j} w_h.$$

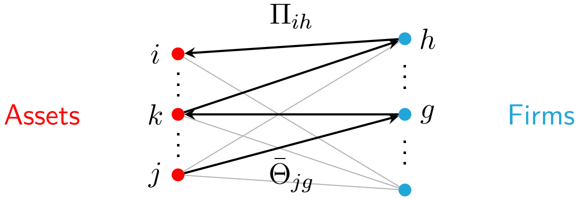
Portfolio tracking: $\Pi =$ constant ... $H \equiv 0$.

Network Effect

$$\left[I - \Pi\bar{\Theta}^\top\right]^{-1} = I + \Pi\bar{\Theta}^\top + \left(\Pi\bar{\Theta}^\top\right)^2 + \left(\Pi\bar{\Theta}^\top\right)^3 + \ldots.$$

$$\begin{aligned}\text{Direct Effect} &= I \cdot \text{[DE]}\\ \text{Primary Network Effect} &= \Pi\bar{\Theta}^\top \cdot \text{[DE]}\\ \text{Secondary Network Effect} &= \left(\Pi\bar{\Theta}^\top\right)^2 \cdot \text{[DE]}\end{aligned}$$

$(\Pi\bar{\Theta}^\top)^t_{ij} = t$-th order impact from asset j to i over paths of length $2t$.

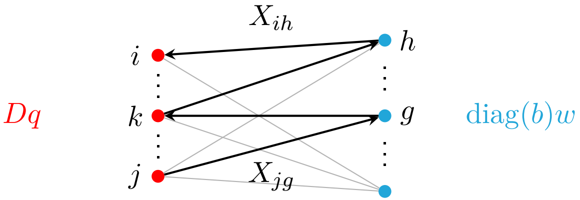


$$\Pi_{ih}\bar{\Theta}_{kh} \cdot \Pi_{kg}\bar{\Theta}_{jg}$$

Network Design

Decompose Π into leverage $b \in \mathbb{R}^{|\mathcal{F}|}$ and holding network $X \in \mathbb{R}^{|\mathcal{A}| \times |\mathcal{F}|}$

$$\begin{aligned} b_h &= \text{leverage for firm } h \\ X_{ih} &= \text{fraction of investment into asset } i \\ \Pi_{ih} &= b_h X_{ih} \end{aligned}$$



holding X

Feasible economies: (D, q, w, b) such that $\mathbf{1}_{\mathcal{A}}^{\top} Dq = w^{\top} b$

Network Design

Set of feasible holding networks

$$\mathcal{X} = \left\{ X : Dq = X \operatorname{diag}(b) w, \mathbf{1}_{\mathcal{A}}^{\top} X = \mathbf{1}_{\mathcal{F}}^{\top}, X \geq 0 \right\}$$

Suppose Θ is in equilibrium. Then

$$\Pi \bar{\Theta}^{\top} = Y(X) \triangleq X \operatorname{diag}(b) \operatorname{diag}(w) \operatorname{diag}(b) X^{\top} [D \operatorname{diag}(q)]^{-1}.$$

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The Maximal Network Amplifier

$$\text{MNA} \triangleq \rho([NE(X)]) = \rho([I - Y(X)]^{-1})$$

where $\rho(\cdot)$ is the spectral radius of a matrix.

Network Design

Set of feasible holding networks

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The Maximal Network Amplifier

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where $\rho(\cdot)$ is the spectral radius of a matrix.

Network design problem: $\min_{X \in \mathcal{X}} \text{MNA}(X)$

Low and High Leverage Regimes

Define: $\underline{\lambda_{\max}} \triangleq \min_{X \in \mathcal{X}} \lambda_{\max}(Y(X))$ $\overline{\lambda_{\min}} \triangleq \max_{X \in \mathcal{X}} \lambda_{\min}(Y(X))$

Low leverage economy $\triangleq \underline{\lambda_{\max}} < 1$ High leverage economy $\triangleq \overline{\lambda_{\min}} > 1$

Low and High Leverage Regimes

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Low leverage economy $\triangleq \underline{\lambda_{\mathsf{max}}} < 1$ High leverage economy $\triangleq \overline{\lambda_{\mathsf{min}}} > 1$

Theorem: For any economy

$$\overline{\lambda_{\mathsf{min}}} \leq \underline{\lambda_{\mathsf{max}}}$$

Low leverage and high leverage economies are disjoint.

Desirable network: Low Leverage Economy

Theorem: For a low-leverage economy:

$$\min_{X \in \mathcal{X}} \mathsf{MNA}(X) \leq \frac{1}{1 - \underline{\lambda_{\mathsf{max}}}}$$

Bound achieved by the mutual-fund network

$$X^* \triangleq \frac{1}{\mathbf{1}_{\mathcal{A}}^\top Dq} (Dq) \mathbf{1}_{\mathcal{F}}^\top$$

In a mutual fund network

- All firms invest in the same portfolio
- The risks of the firms are completely pooled
- Risk management achieved by diversification

Desirable network: High Leverage Economy

Theorem For a high-leverage economy

$$\min_{X \in \mathcal{X}} \mathsf{MNA}(X) \le \frac{1}{\overline{\overline{\lambda_{\mathsf{min}}}} - 1}$$

Bound asymptotically achieved by an isolated network

$$X^* = \begin{pmatrix} 1 & \dots & 1 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & \dots & 0 & 1 & \dots & 1 & 0 & \dots & 0 \\ 0 & \dots & 0 & 0 & \dots & 0 & \dots & \dots & \dots \\ 0 & \dots & 0 & 0 & \dots & 0 & 1 & \dots & 1 \end{pmatrix}$$

- Firms invest in only one asset
- The firms are clustered into groups that do not interact
- Risk management achieved by diversity

Systemic risk management $\equiv$ managing feedback

Systemic risk is a consequence of positive feedback loops

Networks or directed graphs do not enough information to identify them

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Propose signed digraphs (SGD) as the next level of detail

- Used in the process engineering literature
- Extends the analysis from arcs to loops – non-local interactions
- Systematic analysis of the hazards and instabilities
- Compromise between full control theoretic analysis and graphs

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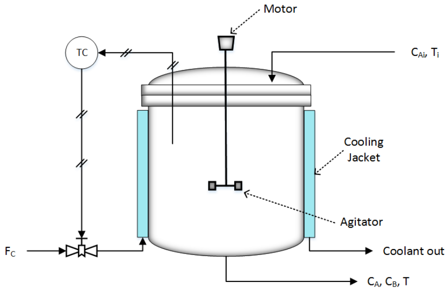
Financial entity $\equiv$ processing plant that transforms inputs to outputs

- Graphs are good for flows, e.g. internet, power grid, etc.
- Signed digraphs are good for flow transformations

SDG example: Continuous stirred reactor

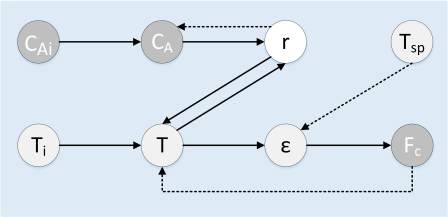


Inputs: Concentration c_{A_i} of A and temperature T_i

Output: Exothermal Reaction $A \rightarrow B$

Control: Temperature set point T_{sp}

SDG example: Continuous stirred reactor

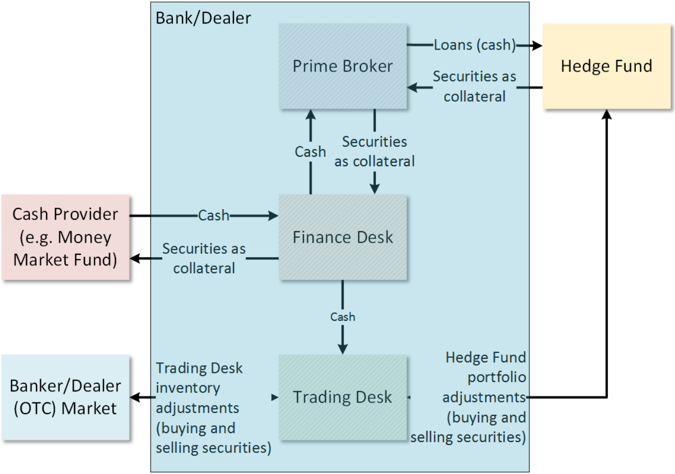


- Solid arc: positive gradient, e.g. $\frac{\partial c_A}{\partial c_{A_i}} > 0$.
- Negative arc: negative gradient, e.g. $\frac{\partial c_A}{\partial r} < 0$.

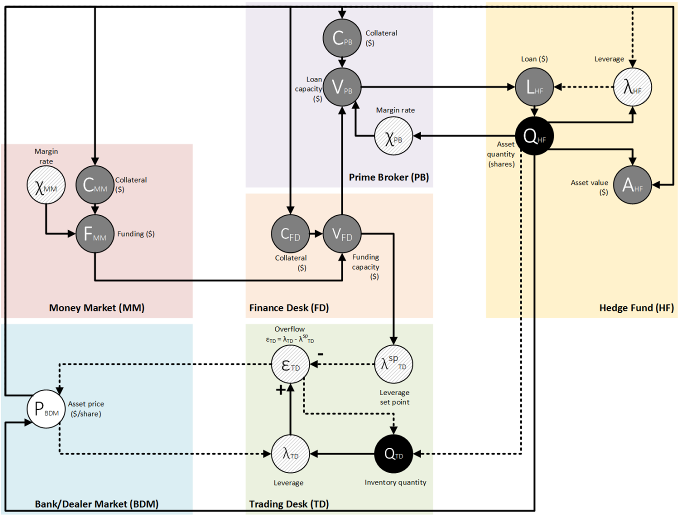
Loops

- $c_A \to r \to c_A$: negative feedback
- $T \to r \to T$: positive feedback
- $T \to \epsilon \to F_c \to T$: negative feedback

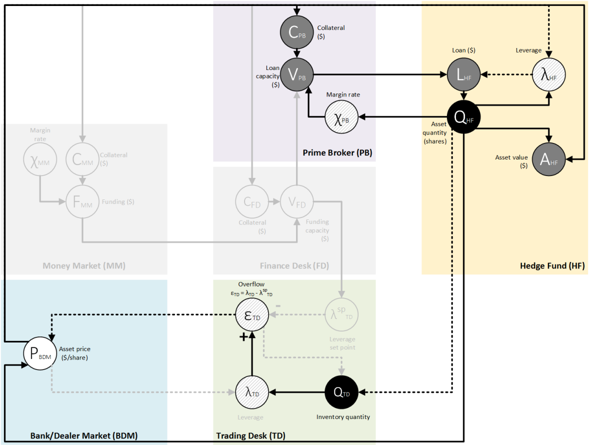
Simplified bank-dealer network



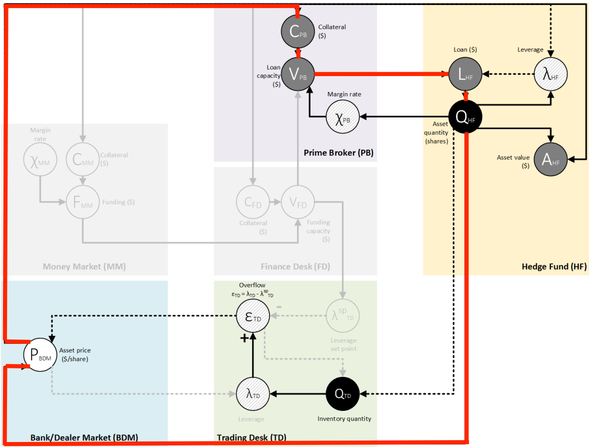
SDG for bank-dealer



SDG for bank-dealer: fire sales



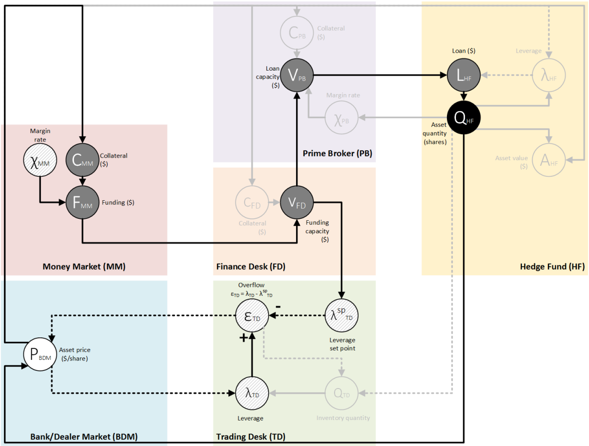
SDG for bank-dealer: fire sales



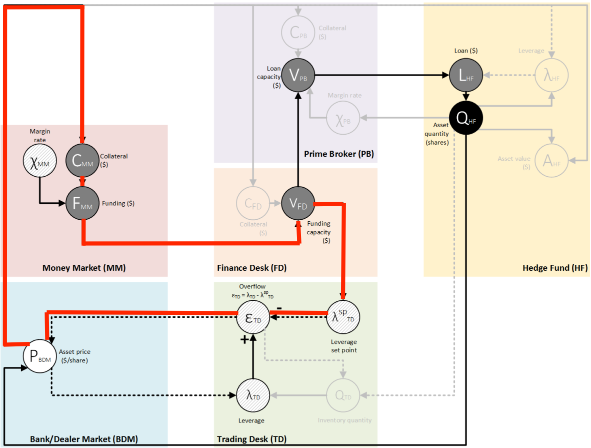
Positive feedback loop

- $P_{BDM} \rightarrow C_{PB} \rightarrow V_{PB} \rightarrow L_{HF} \rightarrow Q_{HF} \rightarrow P_{BDM}$

SDG for bank-dealer: funding runs



SDG for bank-dealer: funding runs



Positive feedback loop

- $P_{BDM} \rightarrow C_{MM} \rightarrow F_{MM} \rightarrow V_{FD} \rightarrow \lambda_{TD}^{SP} \rightarrow \epsilon_{TD} \rightarrow P_{BDM}$

Summary

An axiomatic framework for systemic risk

- Subsumes many recently proposed risk measures
- Structural decomposition of systemic risk
- Methodology extends to a much broader class of risk functions

Structural model for asset-firm contagion

- Endogenous asset prices
- Direct Effect, propagated via Network Effect, forms price change
- low-leverage economies favor mutual fund holding networks
- high-leverage economies favor isolated holding networks

Signed di-graph (SGD) to identify positive feedback loops

- Fast depth first algorithms for discovering loops
- Identifies fire sales and funding runs