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Challenges in the Application of Mathematical Programming Approaches to Enterprise-wide Optimization of Process Industries

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Motivation for Enterprise-wide Optimization

Global chemical/petrochemical industry

Strong international competition process industry
Pressure for reducing costs, inventories and ecological footprint

⇒ **Major goal: Enterprise-wide Optimization**

At interface with Operations Research and Industrial Engineering

A major challenge: optimization models and solution methods

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Enterprise-wide Optimization (EWO)

EWO term coined at FOCAPO-2003 (Marco Duran)

EWO involves optimizing the operations of R&D, material supply, manufacturing, distribution of a company to <u>reduce costs, inventories</u>, <u>ecological footprint</u> and to <u>maximize profits, responsiveness</u> .

Key element: Supply Chain

Example: petroleum industry

Wellhead

Trading

Transfer of Crude

Refinery Processing

Schedule Products

Transfer of Products

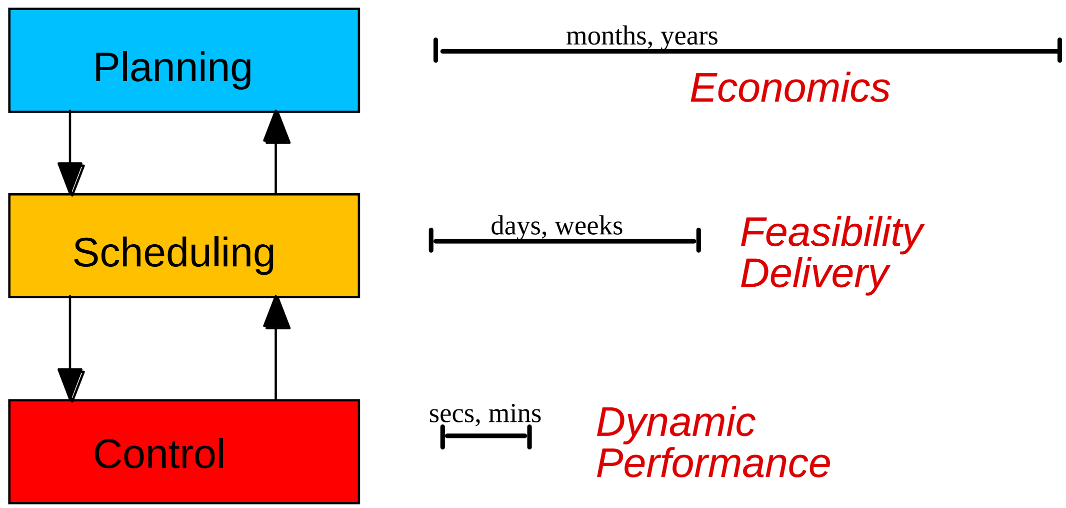
Terminal Loading

Pump

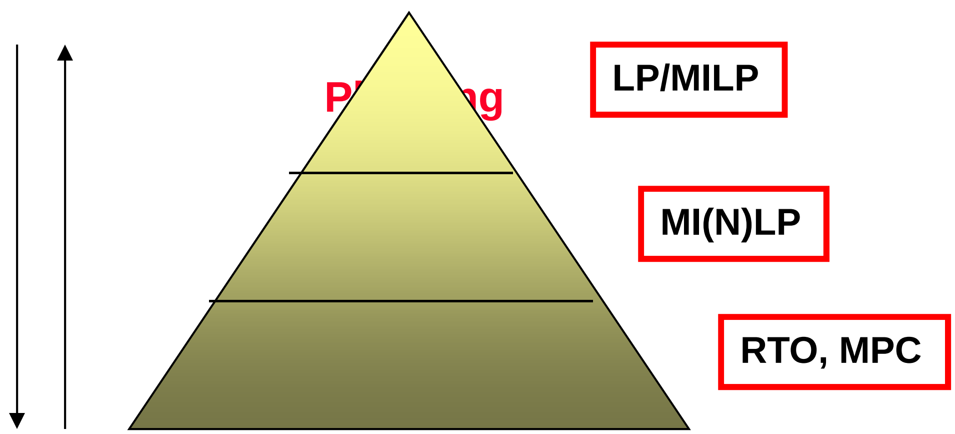
Key issues:

I. Integration of planning, scheduling and control

Multiple time scales



Multiple models



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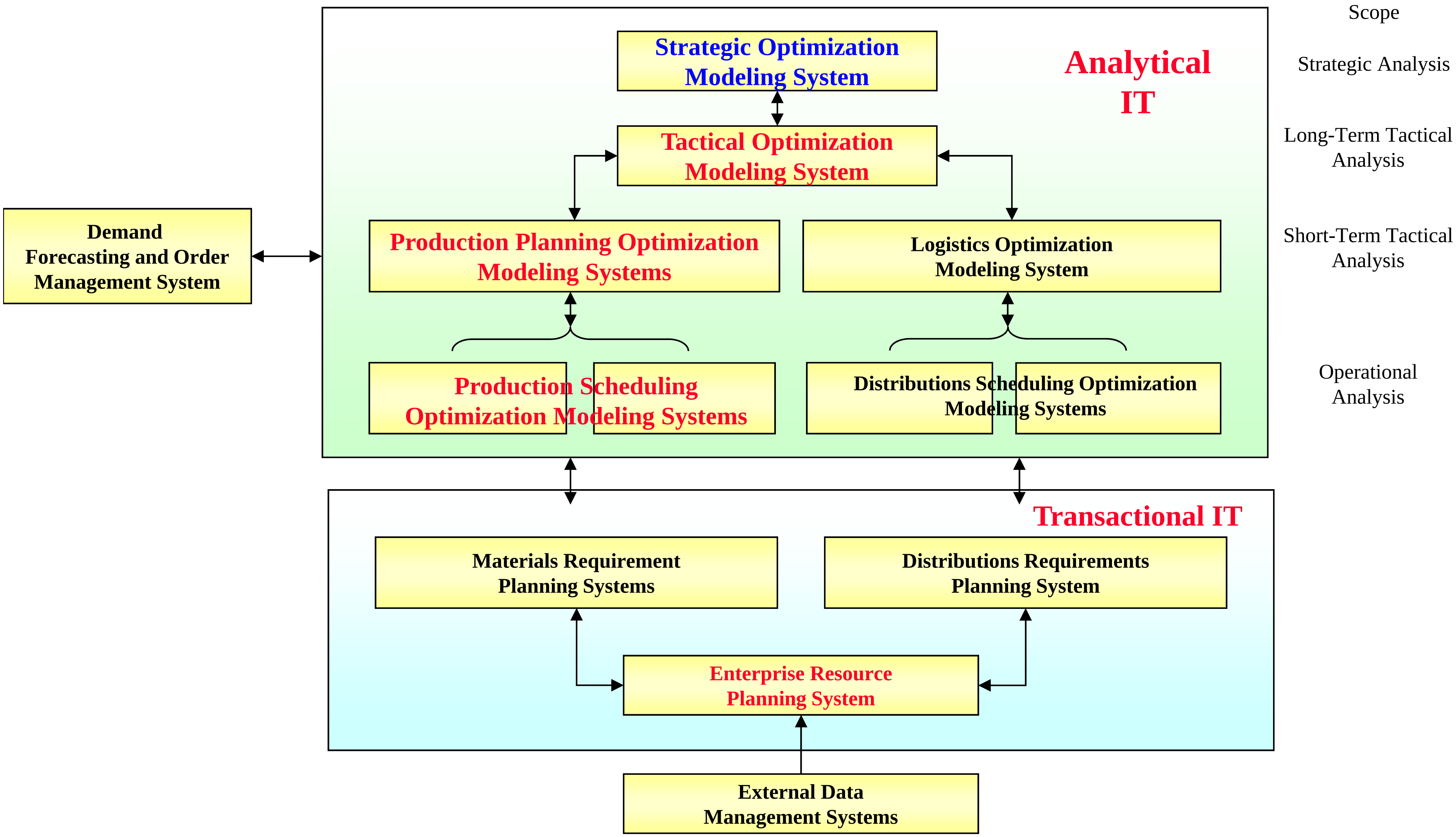
II. Integration of information and models/solution methods



Source: Tayur, et al. [1999]

Optimization Modeling Framework: Mathematical Programming

$$\min Z = f(x, y)$$

Objective function

$$s.t. \quad h(x, y) = 0$$

$$g(x, y) \leq 0$$

Constraints

$$x \in R^n, \quad y \in \{0,1\}^m$$

MINLP: Mixed-integer Nonlinear Programming Problem

Linear/Nonlinear Programming (LP/NLP)

$$\min Z = f(x)$$
$$s.t. \quad h(x) = 0$$
$$g(x) \le 0$$
$$x \in R^n$$

LP Codes:
CPLEX, XPRESS, GUROBI, XA

Very large-scale models
Interior-point: solvable polynomial time

NLP Codes:
CONOPT *Drud (1998)*
IPOPT *Waechter & Biegler (2006)*
Knitro *Byrd, Nocedal, Waltz (2006)*
MINOS *Murtagh, Saunders (1995)*
SNOPT *Gill, Murray, Saunders(2006)*
BARON *Sahinidis et al. (1998)*
Couenne *Belotti, Margot (2008)*
GloMIQO *Misener, Floudas (2012)*
} Global Optimization (BARON, Couenne, GloMIQO)

Large-scale models

Issues:
Convergence
Nonconvexities

Mixed-integer Linear/Nonlinear Programming (*MILP/MINLP*)

$$\min Z = f(x,y)$$
$$s.t. \quad h(x,y) = 0$$
$$g(x,y) \le 0$$
$$x \in R^n, \quad y \in \{0,1\}^m$$

MILP Codes:

CPLEX, XPRESS, GUROBI, XA

Great Progress over last decade

MINLP Codes:

DICOPT (GAMS) *Duran and Grossmann (1986)*
a-ECP *Westerlund and Petersson (1996)*
MINOPT *Schweiger and Floudas (1998)*
MINLP-BB (AMPL) *Fletcher and Leyffer (1999)*
SBB (GAMS) *Bussieck (2000)*
Bonmin (COIN-OR) *Bonami et al (2006)*
FilMINT *Linderoth and Leyffer (2006)*
BARON *Sahinidis et al. (1998)*
Couenne *Belotti, Margot (2008)*
GloMIQO *Misener, Floudas (2012)*

} Global Optimization (BARON, Couenne, GloMIQO)

New codes over last decade leveraging progress in MILP/NLP

Issues:
Convergence
Nonconvexities
Scalability

Modeling systems

Mathematical Programming

GAMS *(Meeraus et al, 1997)*

AMPL *(Fourer et al., 1995)*

AIMSS *(Bisschop et al. 2000)*

1. Algebraic modeling systems => pure equation models
2. Indexing capability => large-scale problems
3. Automatic differentiation => no derivatives by user
4. Automatic interface with
   LP/MILP/NLP/MINLP solvers

Have greatly facilitated development and implementation of Math Programming models

Generalized Disjunctive Programming (GDP)

$$\min Z = \sum_k c_k + f(x)$$

$$s.t. \quad r(x) \leq 0$$

$$\bigvee_{j \in J_k} \begin{bmatrix} Y_{jk} \\ g_{jk}(x) \leq 0 \\ c_k = \gamma_{jk} \end{bmatrix} \quad k \in K$$

$$\Omega(Y) = true$$

$$x \in R^n, \; c_k \in R^1$$

$$Y_{jk} \in \{ true, false \}$$

Raman, Grossmann (1994)

Disjunctions

Logic Propositions

Continuous Variables

Boolean Variables

Codes:
LOGMIP (*GAMS-Vecchietti, Grossmann, 2005*)
EMP (*GAMS-Ferris, Meeraus, 2010*)

Other logic-based: *Constraint Programming (Hooker, 2000)*
Codes: *CHIP, Eclipse, ILOG-CP*

Optimization Under Uncertainty

Multistage Stochastic Programming

Birge & Louveaux, 1997; Sahinidis, 2004

$$min\ \ z = c^1x^1 + E_{\omega^2}[c^2(\omega)x^2(\omega^2) + \ldots + E_{\omega^N}[c^N(\omega)x^N(\omega^N)]\ldots]$$

s.t.

$$W^1x^1 = h^1$$

$$T^1(\omega)x^1 + W^2x^2(\omega^2) = h^2(\omega)$$

$$\vdots$$

Exogeneous uncertainties
(e.g. demands)

$$T^{N-1}(\omega)x^{N-1}(\omega^{N-1}) + W^Nx^N(\omega^N) = h^N(\omega)$$

$$x^1 \geq 0, x^t(\omega^t) \geq 0, t = 2,\ldots, N-1$$

Special case: two-stage programming (N=2)

x^1 stage 1 ω ***x^2 recourse (stage 2)***

Here and now ***Wait and see***

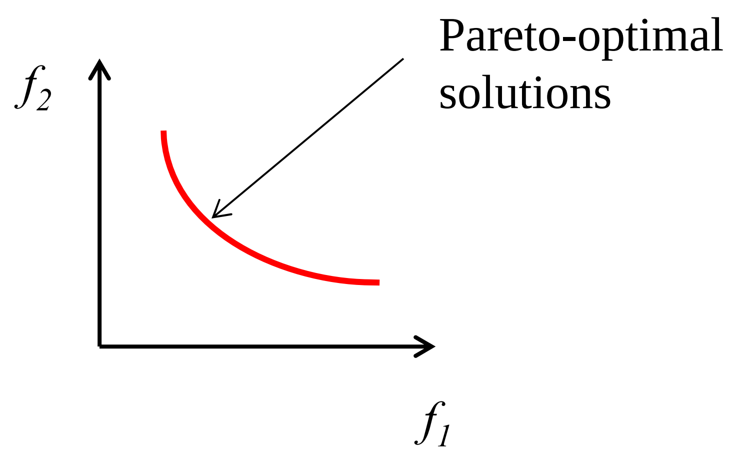
Multiobjective Optimization

$$\min Z = \begin{bmatrix} f_1(x,y) \\ f_2(x,y) \\ \cdots\cdots \end{bmatrix}$$

$$s.t. \quad h(x,y) = 0$$

$$g(x,y) \leq 0$$

$$x \in R^n, \quad y \in \{0,1\}^m$$



ε-constraint method: Ehrgott (2000)

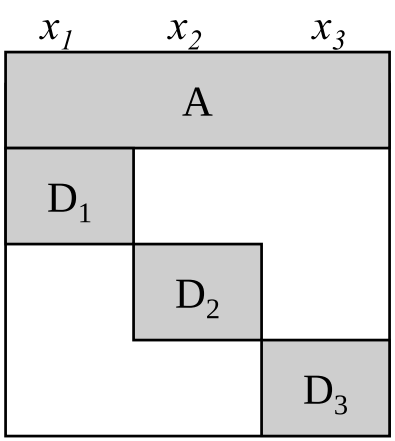
Parametric programming: Pistikopoulos, Georgiadis and Dua (2007)

Decomposition Techniques

Lagrangean decomposition

Geoffrion (1972) Guinard (2003)

Complicating Constraints



complicating constraints

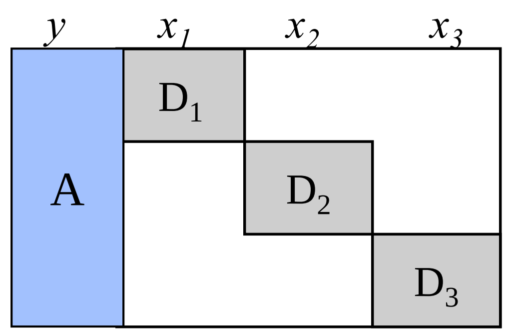
$$\max\ c^T x$$
$$st\ \ Ax = b$$
$$D_i x_i = d_i \ \ i = 1,..n$$
$$x \in X = \{x \,|\, x_i\,, i = 1,..n, |\, x_i \geq 0\}$$

Widely used in EWO

Benders decomposition

Benders (1962), Magnanti, Wing (1984)

Complicating Variables



complicating variables

$$\max\ a^T y + \sum_{i=1,..n} c_i^T x_i$$
$$st\ \ Ay + D_i x_i = d_i \ \ i = 1,..n$$
$$y \geq 0,\ x_i \geq 0, i = 1,..n$$

Applied in 2-stage Stochastic Programming

Institute for Complex Engineered Systems

PITA Project

Special industrial interest group in CAPD:
"Enterprise-wide Optimization for Process Industries"

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http://egon.cheme.cmu.edu/ewocp/

Multidisciplinary team:

Chemical engineers, Operations Research, Industrial Engineering

Researchers:

Carnegie Mellon:
- Ignacio Grossmann (ChE)
- Larry Biegler (ChE)
- Nick Sahinidis (ChE)
- John Hooker (OR)
- Nick Secomandi (Optns Mgmt)

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Overall Goal:

- *Novel planning and scheduling models, including consideration of uncertainty*
- *Effective integration of Production Planning, Scheduling and Real–time Optimization*
- *Optimization of Entire Supply Chains*

ABB, AIR PRODUCTS, AIR LIQUIDE, Aurubis, Braskem, DOW, ExxonMobil, BR PETROBRAS, P&G, PRAXAIR, SK innovation, TOTAL

EWO Projects and case studies with partner companies

ABB: *Integrating RTN scheduling models with ISA-95 standard*
Contact: Iiro Harjunkoski — *Ignacio Grossmann, Pedro Castro*

Air Liquide: *Optimal Production of Industrial Gases with Uncertain Energy Prices*
Contact: Ajit Gopalakrishnan, Irene Lotero, Brian Besancon — *Ignacio Grossmann, Carlos Mendez, Natalia Basan*

Aurubis: *Optimal Scheduling for Copper Concentrates Operations*
Contact: Pablo Garcia-Herreros, Bianca Springub — *Ignacio Grossmann, Brenno Menezes, Ynkkai Song*

Braskem: *Dynamic Optimization of Polypropylene Reactors*
Contact: Rita Majewski, George Ostace — *Larry Biegler, Bobby Balsom*

Dow: *Solution Strategies for Dynamic Warehouse Location under Discrete Transportation Costs*
Contact: John Wassick, Anshul Agrawal, Matt Bassett — *Ignacio Grossmann, Braulio Brunaud*

Dow: *Optimization Models for Reliability-based Turnaround Planning Integrated Sites*
Contact: Satyajith Amaran, Scott Bury, J. Wassick — *Nick Sahinidis, Sreekanth Rajagopalan*

Dow: *Parameter Estimation and Model Discrimination of Batch Solid-liquid Reactors*
Contact: Mukund Patel, John Wassick — *Larry Biegler, Yajun Wang*

ExxonMobil: *Optimal Design and Planning of Electric Power Networks*
Contact: A. Venkatesh, D. Mallapragada, D. Papageorgiou — *Ignacio Grossmann, Cristiana Lara*

Mitsubishi E: *Optimization circuitry arrangements for heat exchangers*
Contact: Christopher Laughman, Arvind U. Raghunathan — *Nick Sahinidis, Nick Ploskas*

P&G: *Kinetic Model Parameter Estimation for Product Stability* *Contact: Ben Weinstein*
Larry Biegler, Mark Daichendt

Praxair: *Mixed-Integer Programming Models for Optimal Design of Reliable Chemical Plants*
Contact: Jose Pinto, Sivaraman Ramaswam — *Ignacio Grossmann, Yixin Ye*

Praxair: *Robust Tactical Planning of Multi-period Vehicle Routing under Customer Order Uncertainty*
Contact: Jose Pinto, Arul Sundramoorthy — *Chrysanthos Gounaris, Anirudh Subramanyam*

SKInnovation: *Complex Crude-oil Refinery Scheduling Optimization*
Contact: Faram Engineer — *Ignacio Grossmann, Brenno Menezes*

Total: *Integration of Reservoir Modeling with Oil Field Planning and Infrastructure Optimization*
Contact: Meriam Chebre — *Ignacio Grossmann, KinshukVerma*

Major Issues

- The multi-scale optimization challenge

- Linear vs Nonlinear models

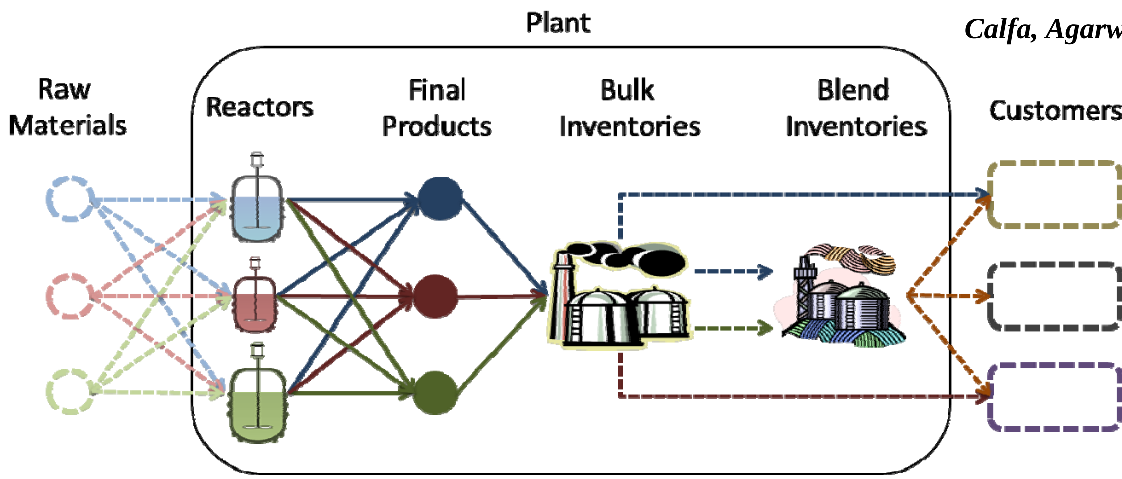
- The uncertainty challenge

- Economics vs. performance

The multi-scale optimization challenge

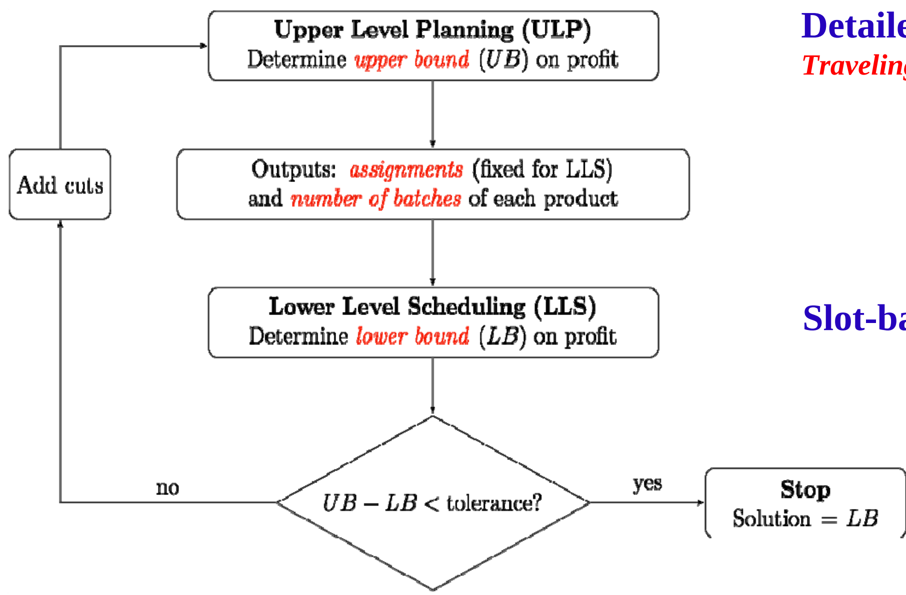
Integrated Planning and Scheduling Batch Plant

Calfa, Agarwal, Wassick, Grossmann (2013)



- Batch units operating in parallel
- Sequence-dependent changeovers between products groups
- A subset of products are blended

Bi-level Decomposition



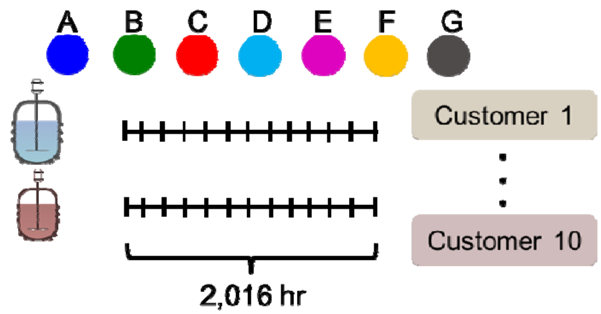
Detailed MILP Planning

Traveling-salesman constraints

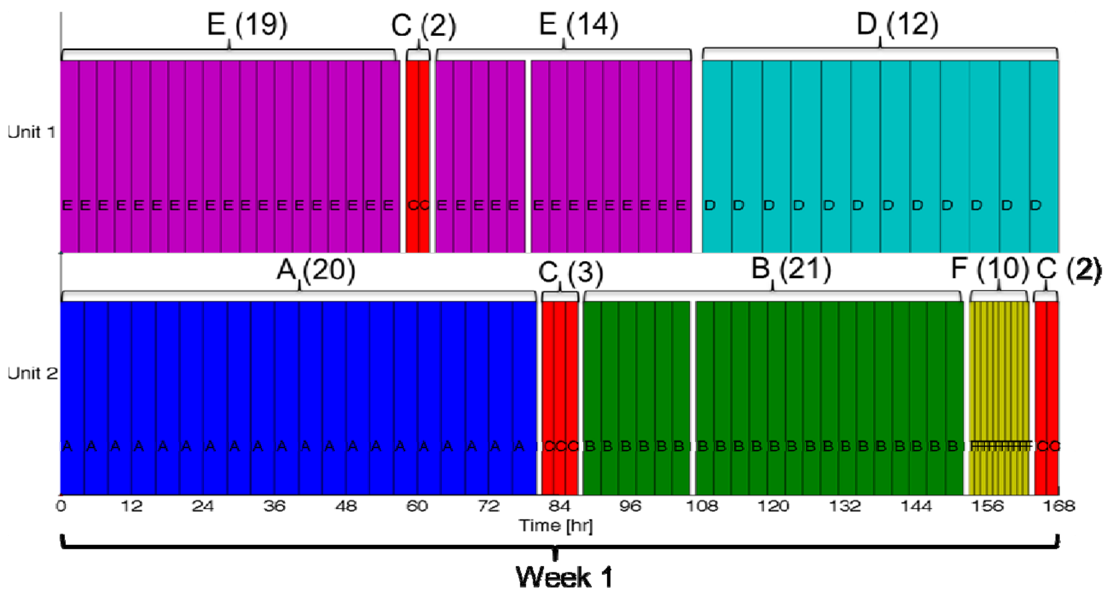
Slot-based MILP model

Example

2 parallel units
2 raw materials
7 products (6 individual and 1 blended)
10 customers
Time horizon
12 weeks



Optimal Schedule (week 1)



Bilevel decomposition converged in one iteration!

Upper level MILP: **1,032 0-1 1,800 cont.v. 3,300 constr. 2.5 sec**

Lower level MILP: **19,600 0-1 23,100 cont.v. 15,300 constr 479 sec**

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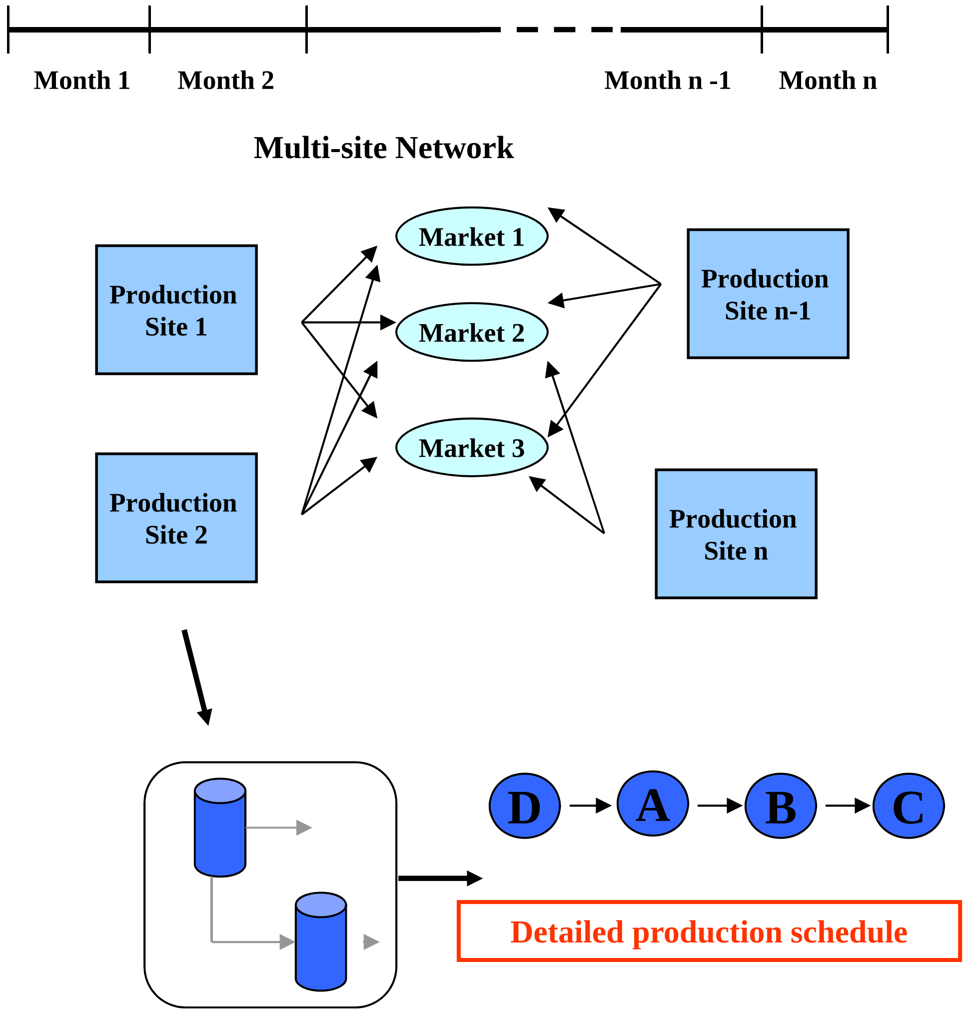
Multisite planning and scheduling

Terrazas, Grossmann (2011)



Given:

- **Network of production sites and markets**
- **Production rates**
- **Forecast demands over time horizon**
- **Sequence-dependent transition times**
- **All costs and product prices**
- **Shipment and storage max capacities**

Determine:

- **Product assignment to sites**
- **Production sequencing**
- **Detailed production scheduling**
- **Production and shipment volumes**
- **Inventory levels**

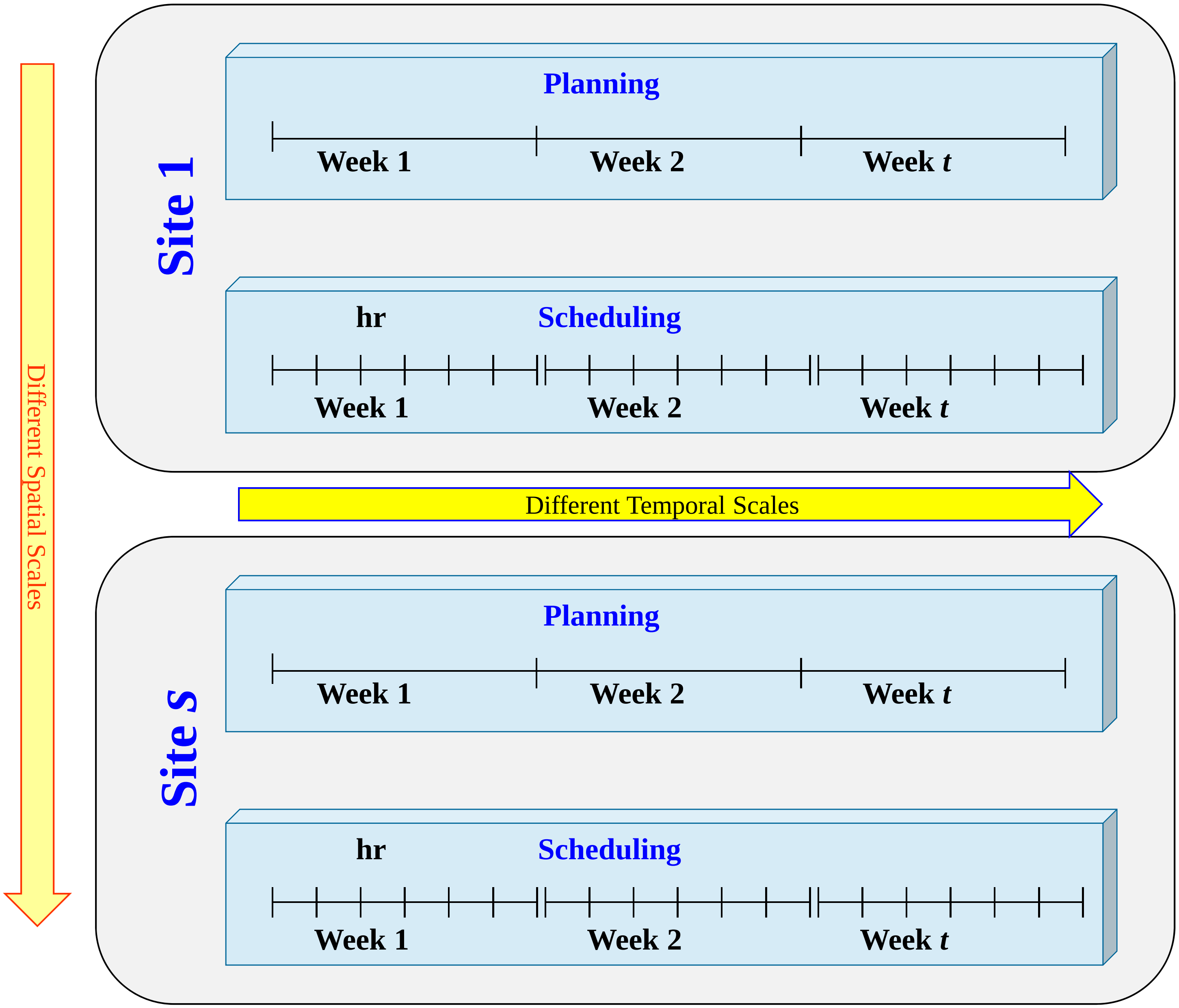
Objective: Maximize profit

Profit = Sales

- Operating costs - Inventory costs

- Distribution costs- Changeover costs

Multi-site planning and scheduling involves different temporal and spatial scales



Weekly aggregate production:

- Amounts
- Aggregate sequencing model
  (TSP constraints)

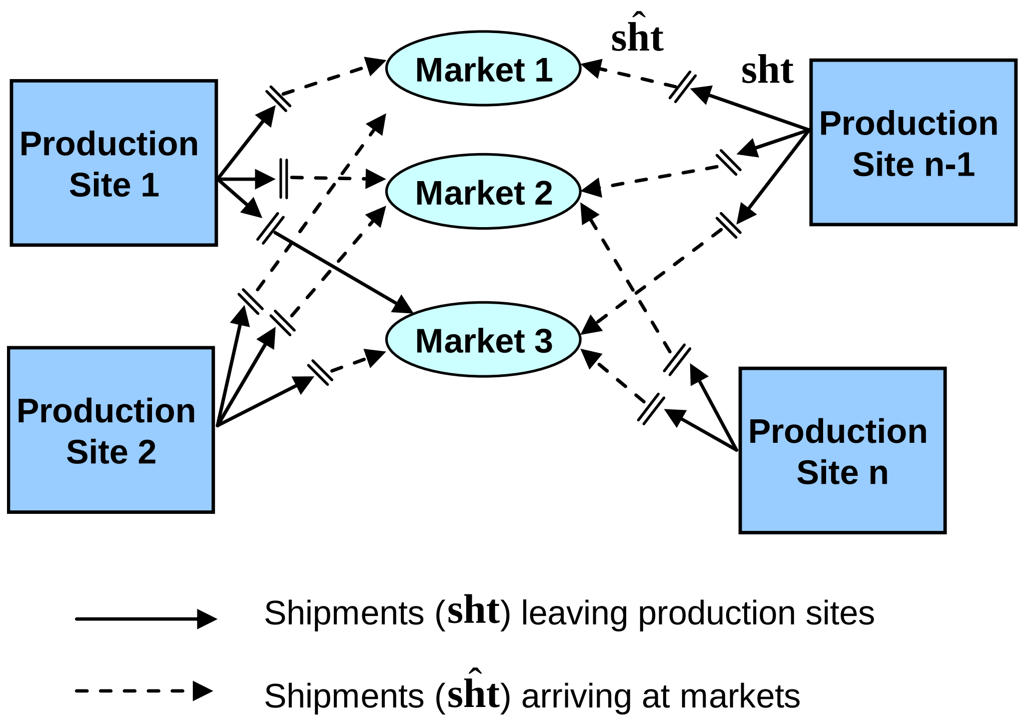
Detailed operation

- Start and end times
- Allocation to parallel lines

Weekly aggregate production:

Detailed operation

Bilevel decomposition + Lagrangean decomposition



- **Bilevel decomposition**
  - Decouples planning from scheduling
  - Integrates across temporal scale
- **Lagrangean decomposition**
  - Decouples the solution of each production site
  - Integrates across spatial scale

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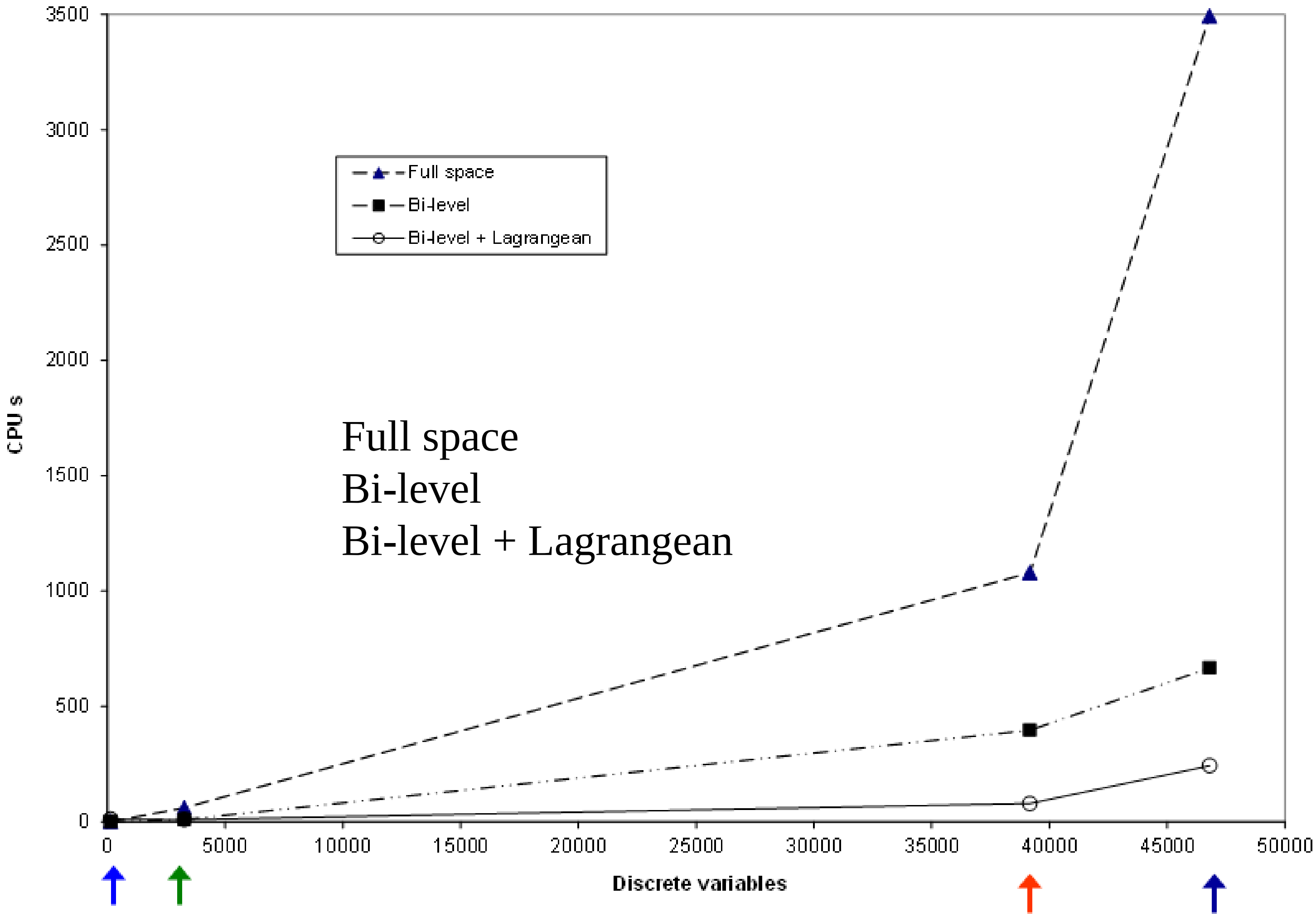
Large-scale problems



3 sites
2 markets
3 products
4 weeks

3 sites
2 markets
16 products
4 weeks

6 sites
6 markets
16 products
24 weeks

6 sites
6 markets
25 products
12 weeks

Optimal Multi-scale Capacity Planning under Hourly Varying Electricity Prices

Mitra, Grossmann, Pinto, Arora (2012)

Price forecast

Given:

- Power-intensive plant
- Products $g \in G$ (Storable and Nonstorable)
- Product demands d_g^t for season $t \in T$
- Seasonal electricity prices on an hourly basis $e^{t,h}$, $t \in T$, $h \in H$
- Upgrade options $u \in U$ of existing equipment
- New equipment options $n \in N$
- Additional storage tanks $st \in ST$

Determine:

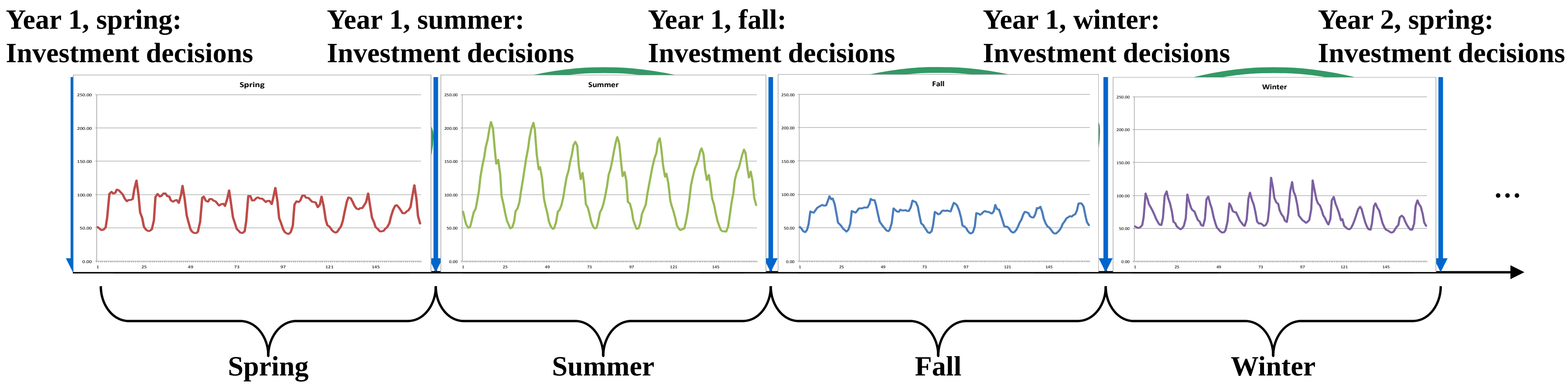
- Production levels $Pr_g^{t,h}$
- Mode of operation $\tilde{y}_{m,o}^{t,h}$, $y_m^{t,h}$
- Sales $S_g^{t,h}$
- Inventory levels $INV_g^{t,h}$

for each season on an hourly basis

- Upgrades for equipment $VU_{m,u}^t$
- Purchase of new equipm. VN_n^t
- Purchase of new tanks $VS_{st,g}^t$

With minimum investment and operating costs

Incorporating design decisions: seasonal variations drive the development of a seasonal model



- Horizon: 5-15 **years**, each year has 4 **periods** (spring, summer, fall, winter)
- Each period is represented by **one week on an hourly basis**
- Each representative week is repeated in a **cyclic** manner (**13** weeks reduced to **1** week) (8736 hr vs. 672 hr)
- Design decisions are modeled by **discrete equipment sizes**

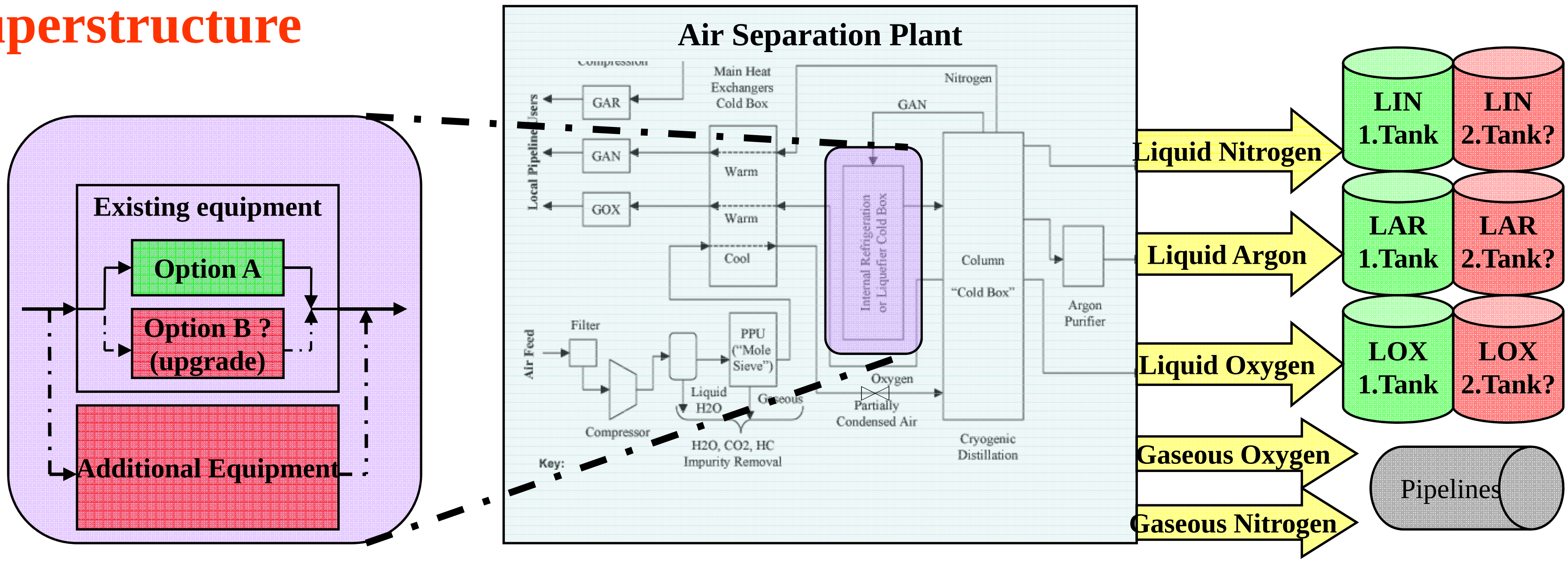
MILP model for multi-scale capacity planning

min $OBJ = \sum_t (Cost_{ops}^t + Cost_{invest}^t) \quad (37)$

Operational Disjunction over the modes that describe the feasible region	Operational Logic constraints for transitions (e.g. minimum uptime/downtime)	Operational Mass balances for inventory, constraints related to demand
Strategic Additional storage Strategic Additional equipment Idea: additional modes for which variables are controlled by the corresponding binary investment	Strategic Equipment replacement Idea: the corresponding mode has an alternative feasible region	Terms for the objective functi

Retrofitting an air separation plant

Superstructure



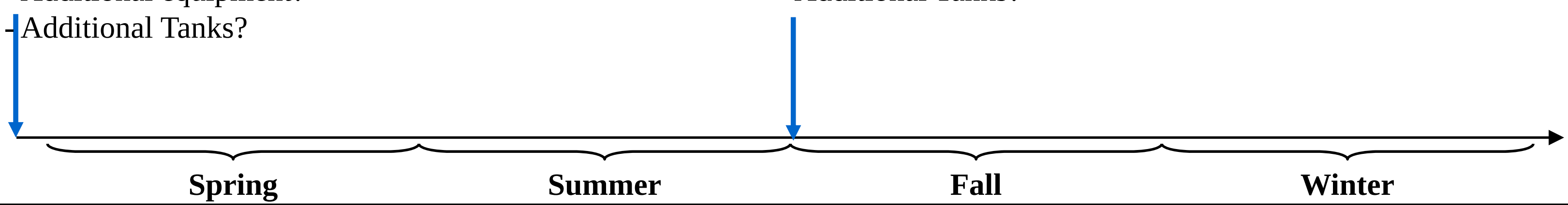
Time

Spring - Investment decisions: (yes/no)

- Option B for existing equipment?
- Additional equipment?
- Additional Tanks?

Fall - Investment decisions: (yes/no)

- Option B for existing equipment?
- Additional equipment?
- Additional Tanks?



- The resulting MILP has **191,861 constraints** and **161,293 variables (18,826 binary.)**
- Solution time: **38.5 minutes** (GAMS 23.6.2, GUROBI 4.0.0, Intel i7 (2.93GHz) with 4GB RAM

Investments increase flexibility help realizing savings.

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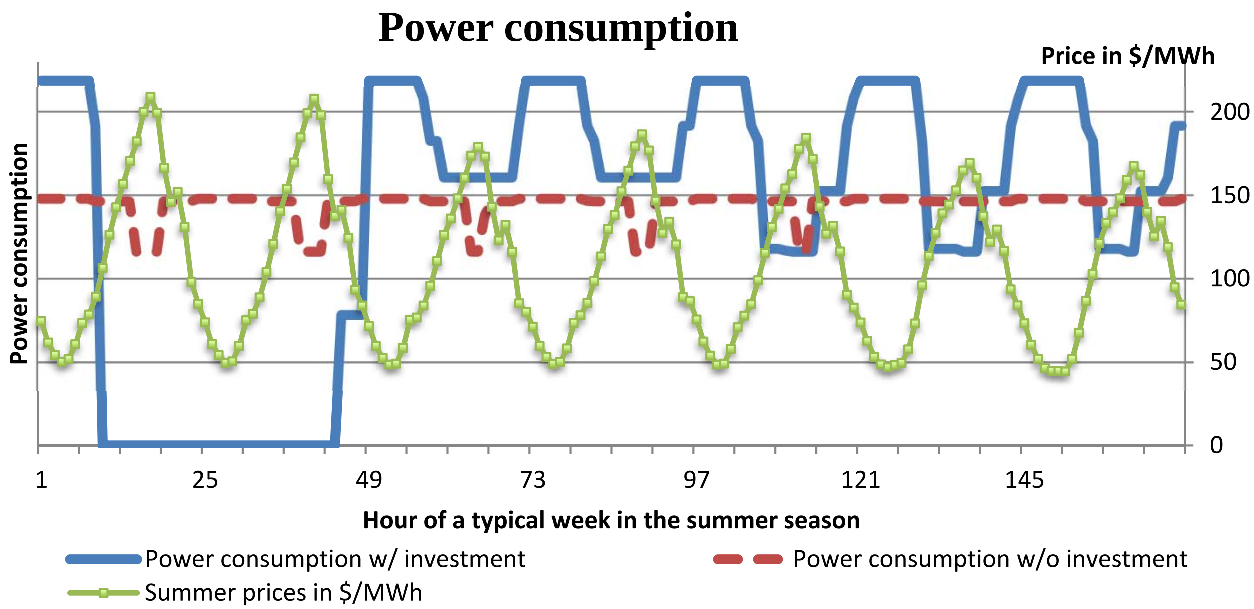
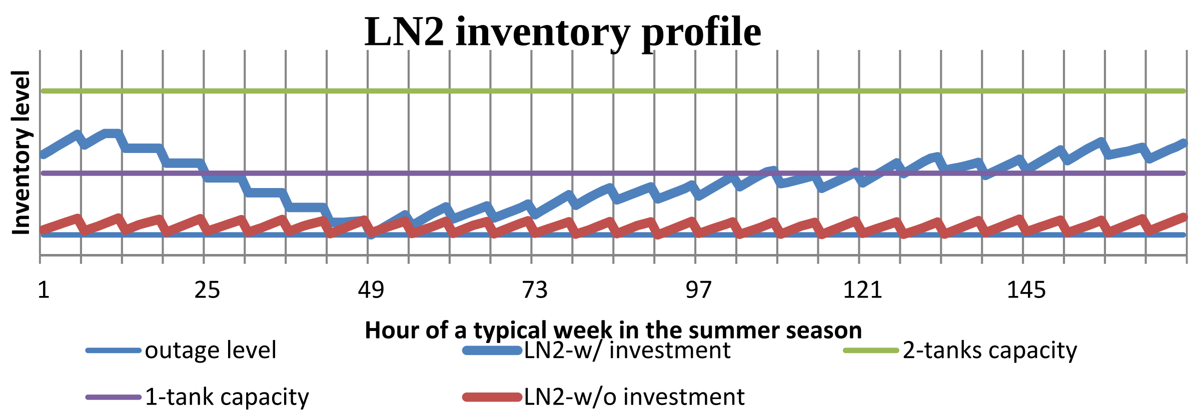




Remarks on case study

- **Annualized costs: $5,700k/yr**
- **Annualized savings: $400k/yr**

- Buy new liquefier in the first time period (annualized investment costs: $300k/a)
- Buy additional LN2 storage tank ($25k/a)
- Don't upgrade existing equipment ($200k/a) equipment: 97%.

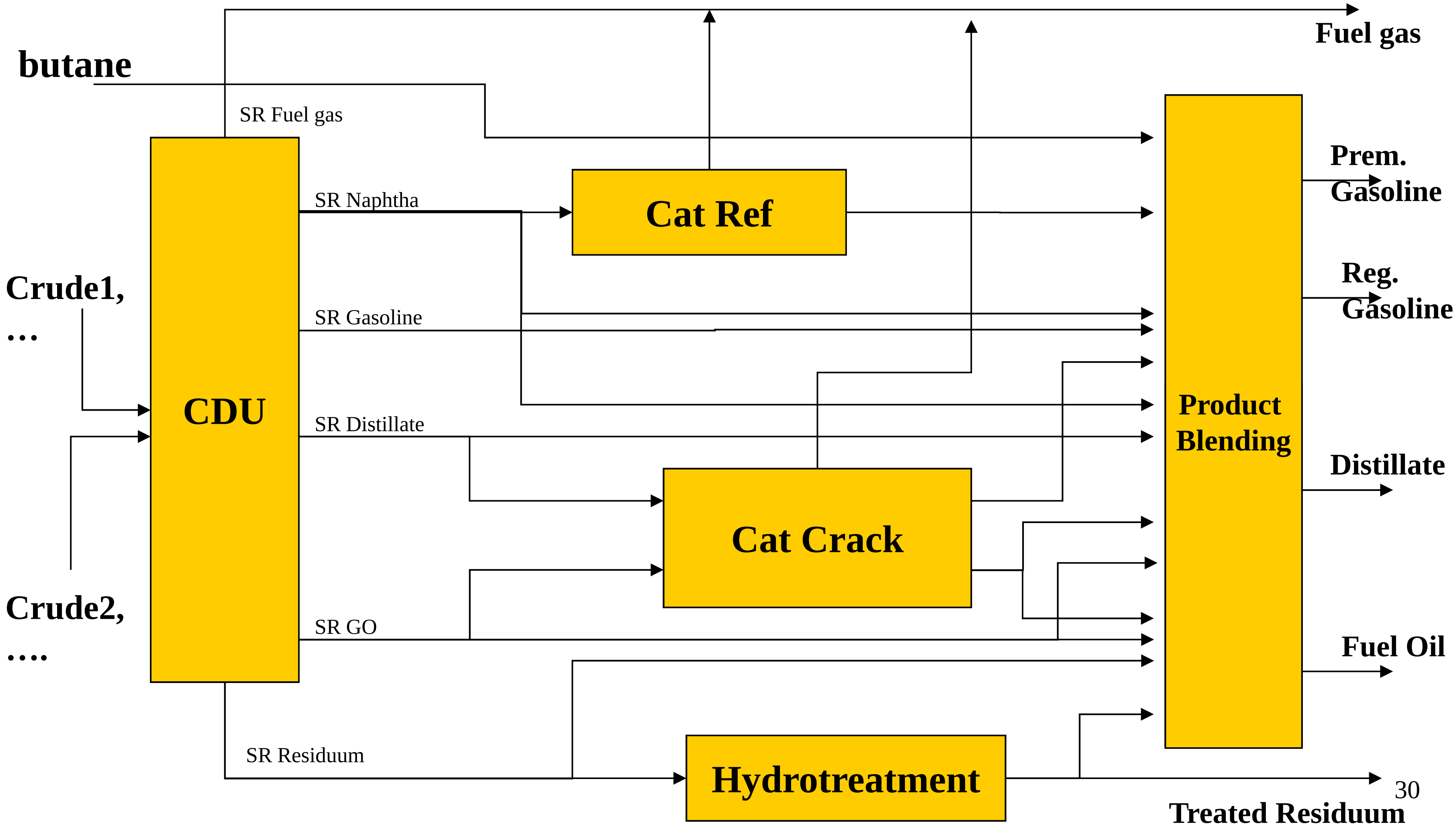
Source: CAPD analysis; Mitra, S., I.E. Grossmann, J.M. Pinto and Nikhil Arora, "Integration of strategic and operational decision- making for continuous power-intensive processes", submitted to ESCAPE, London, Juni 2012

Linear vs. Nonlinear

Nonlinear CDU Models in Refinery Planning Optimization

Alattas, Palou-Rivera, Grossmann (2010)

Typical Refinery Configuration *(Adapted from Aronofsky, 1978)*



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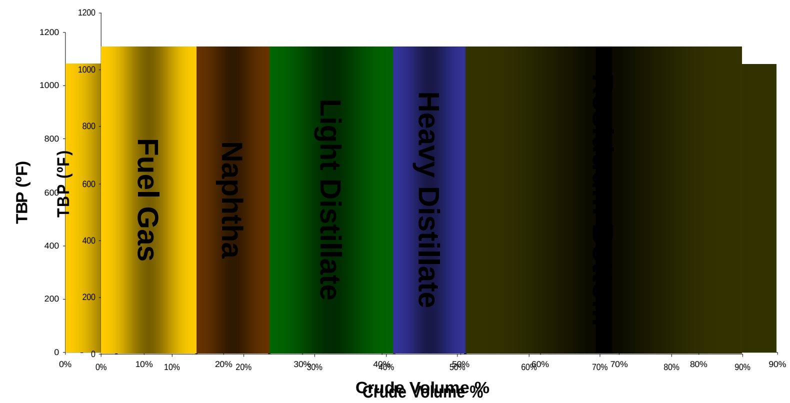
Refinery Planning Models

LP planning models

Fixed yield model
Swing cuts model



Nonlinear FI Model *(Fractionating Index)*

- **FI Model is crude independent**
  - ***FI values are characteristic of the column***
  - ***FI values are readily calculated and updated from refinery data***
- **Avoids more complex, nonlinear modeling equations**
- **Generates cut point temperature settings for the CDU**
- **Adds few additional equations to the planning model**

Planning Model Example Results

Crude1	Louisiana	Sweet	Lightest
Crude2	Texas	Sweet	↓
Crude3	Louisiana	Sour	
Crude4	Texas	Sour	Heaviest

- Comparison of ***nonlinear fractionation index (FI)*** with the fixed yield (FY) and swing cut (SC) models
- Economics: maximum profit

FI yields highest profit

Model	Case1	Case2	Case3
FI	**245**	**249**	**247**
SC	**195**	**195**	**191**
FY	**51**	**62**	**59**

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Model statistics LP vs NLP

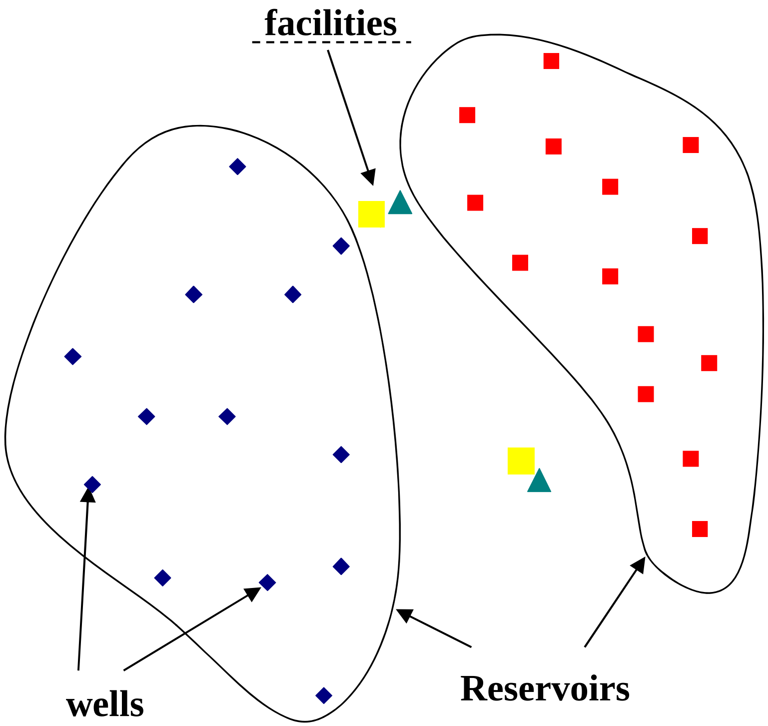
- FI model larger number of equations and variables
- Impact on solution time
- ~30% nonlinear variables

	Model	Variables	Equations	Nonlinear Variables	CPU Time	Solver
2 Crude Oil Case	FY	128	143		0.141	CPLEX
	SC	138	163		0.188	
	FI	1202	1225	348	0.328	CONOPT
3 Crude Oil Case	FY	159	185		0.250	CPLEX
	SC	174	215		0.281	
	FI	1770	1808	522	0.439	CONOPT
4 Crude Oil Case	FY	192	231		0.218	CPLEX
	SC	212	271		0.241	
	FI	2340	2395	696	0.860	CONOPT

Optimal Development of Oil Fields *(deepwater)*

Offshore field having several reservoirs (oil, gas, water)

Gupta, Grossmann (2011)



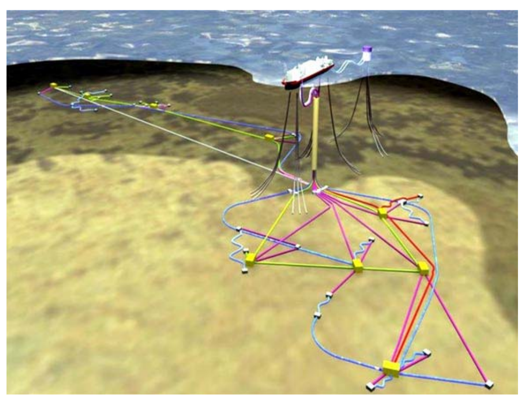
Decisions:

- Number and capacity of FPSO facilities
- Installation schedule for facilities
- Number of sub-seawells to drill
- Oil/gas production profile over time

Objective:

- **Maximize the Net Present Value (NPV) of the project**

FPSO ***(Floating Production Storage Offloading)***



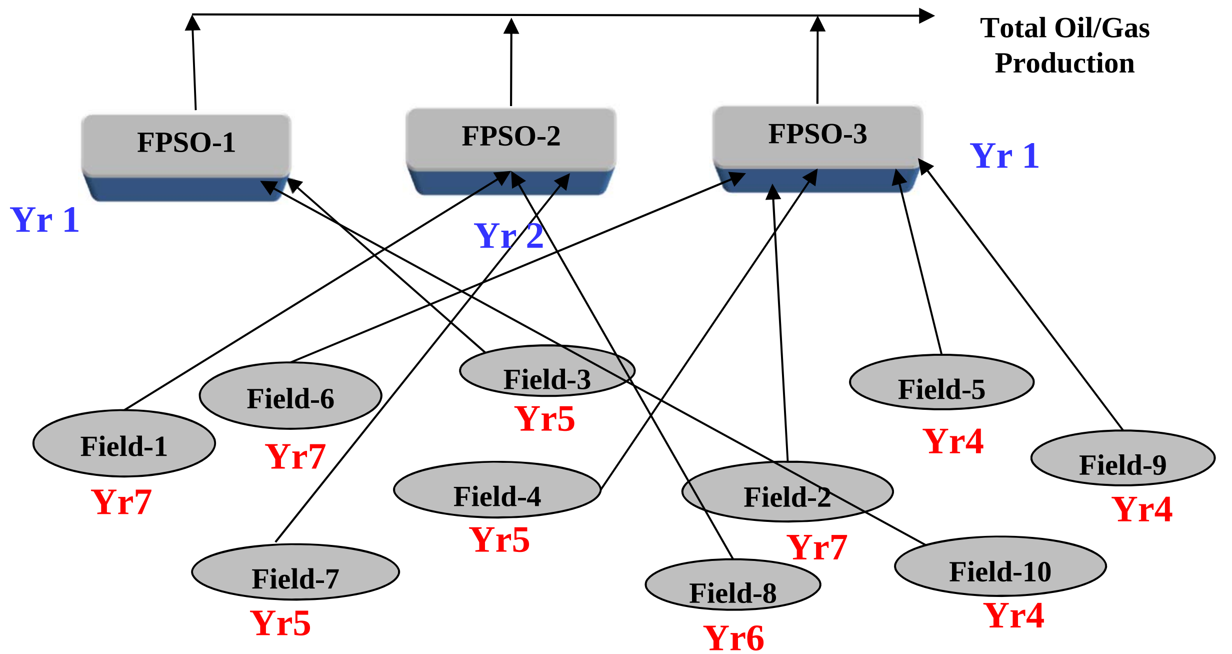
MINLP model

- **Nonlinear reservoir behavior**
- **Three components (oil, water, gas)**
- **Lead times for FPSO construction**
- **FPSO Capacity expansion**
- **Well Drilling Schedule**

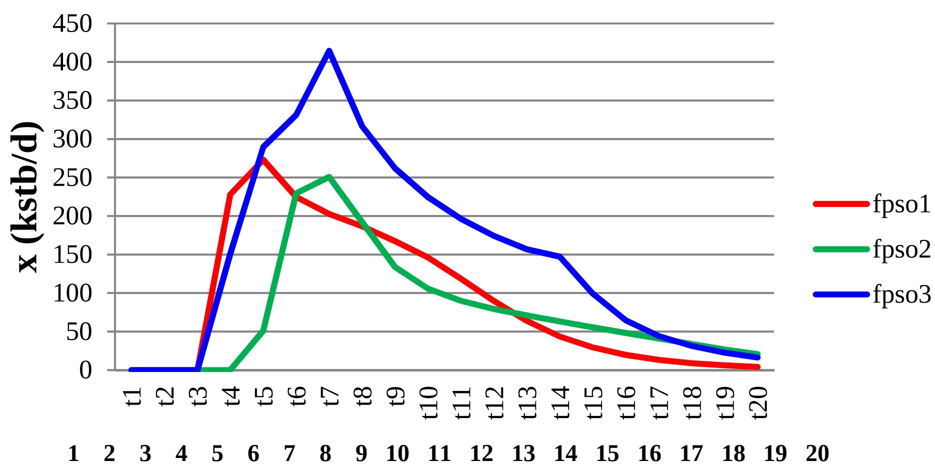
Example

Optimal NPV = *$30.946 billion*

- 20 Year Time Horizon
- 10 Fields
- 3 FPSOs
- 23 Wells
- 3 Yr lead time FPSO
- 1 Yr lead time expansion



Oil Flowrate



	MINLP	Reformulated MILP
Discrete Var.	483	863
SOS1 Var.	0	800
Continuous Var.	5,684	12,007
Constraints	9,877	17,140
Solver	DICOPT 2x-C	CPLEX 12.2
NPV (billion dollars)	30.946	30.986 (<10% gap)
CPU time(s)	67	16,295

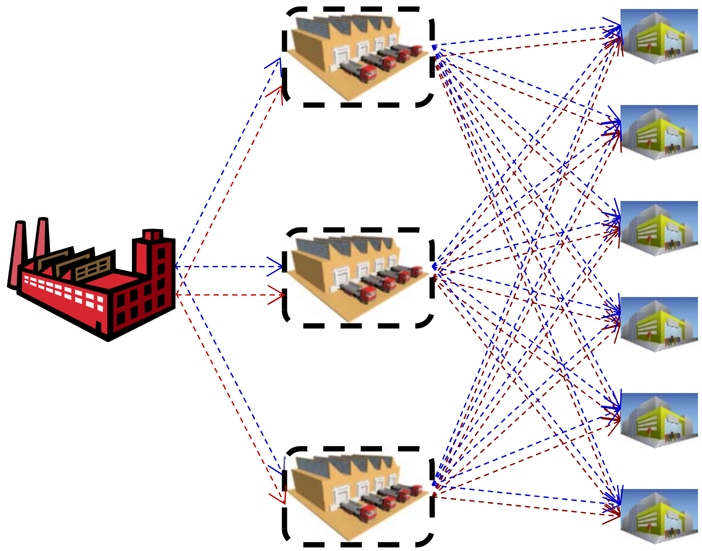
The uncertainty challenge

Resilient Supply Chain Design

Garcia-Herreros, Grossmann, Wassick (2014)

Given:

- Reliable plant
- Candidate locations for DCs with *risk of disruption*
- Set of customer with deterministic demands for multiple commodities
- Set of *scenarios and their associated probabilities*



Minimize cost by:

- Selecting **DCs locations**
- Determining **storage capacity for each commodity** in selected DCs
- **Allocating demands in every scenario**

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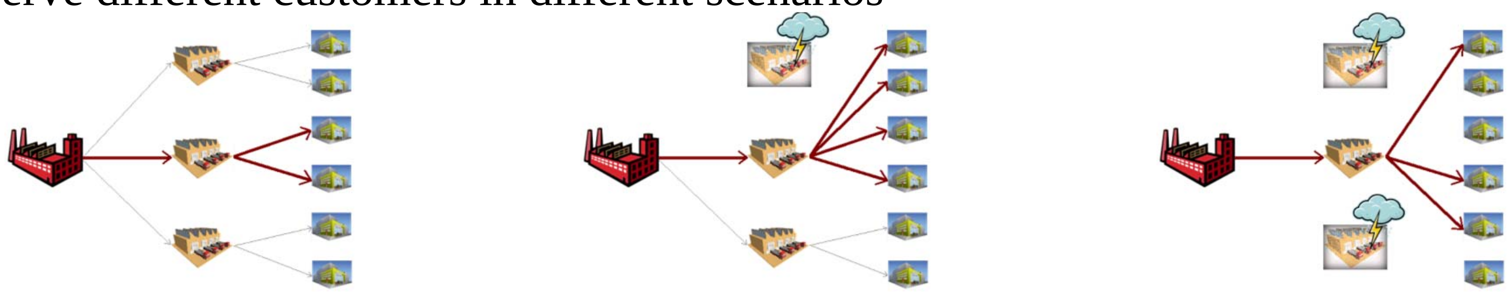
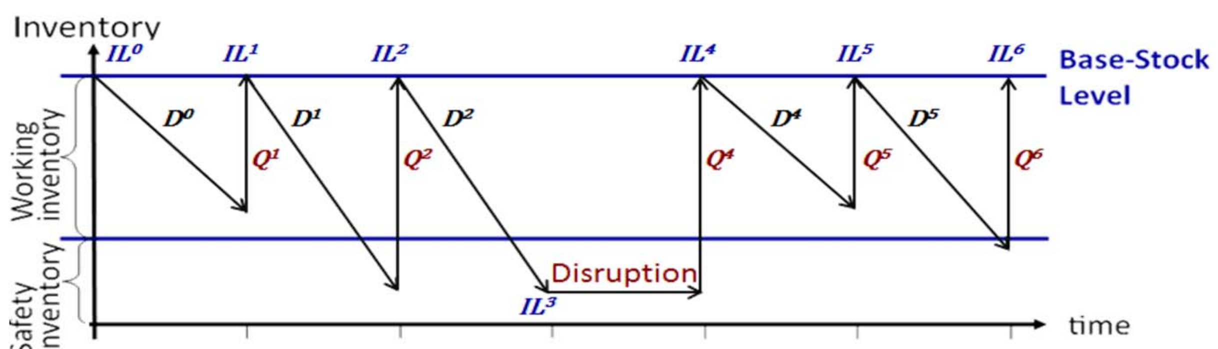
Distribution Strategy

Disruptions give rise to scenarios

DCs serve different customers in different scenarios



Rerouting produces a stochastic demand on DCs



Base-Stock policy with no lead time under disruptions risk

1. Snyder L. V.; Shen, Z-J. M. *Fundamentals of Supply Chain Theory*; John Wiley & Sons: Hoboken (NJ), 2011. pp. 63-116.

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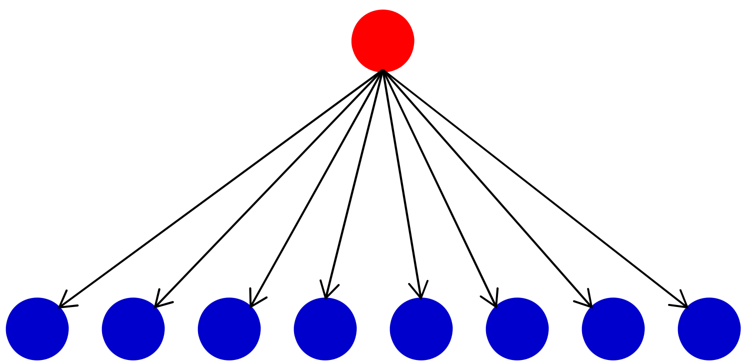
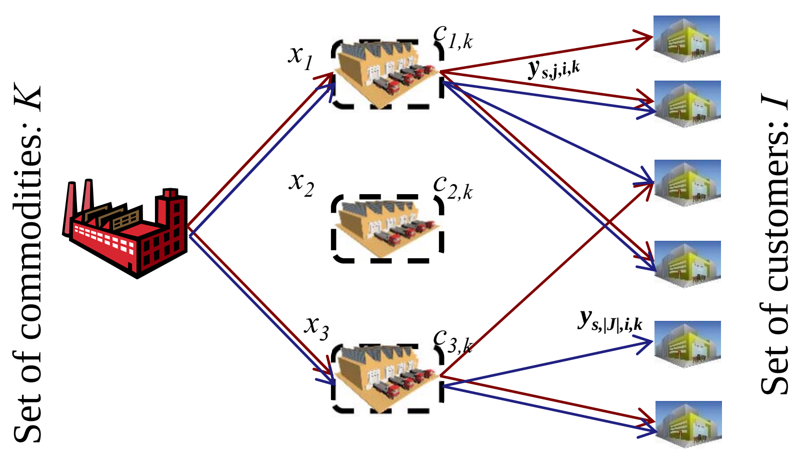
Two-Stage Stochastic Programming



Set of scenarios: S

First stage decisions:

- **DC selection:** $x_j \in \{0,1\}$
- **DC capacities:** $c_{j,k} \in R^+$



Set of DC candidate locations: J

Second stage decisions:

- **Demand allocation:** $y_{s,j,i,k} \in R^+$
- **Penalties:** $y_{s,|J|,i,k} \in R^+$

Objective:

- ***Investment cost***
- ***Expected cost of distribution***

1. Birge, J.; Louveaux, F. V. Introduction to stochastic programming (2nd Ed.); Springer: New York (NY), 2011.

Industrial Instance

Supply chain network optimization:

- 1 Production plant
- 29 candidate DCs with disruption probabilities between 0.5% and 3%
- 110 customers
- 61 commodities

Total number of scenarios: **$2^{29} \approx$ 537 million**

		Deterministic problem	Reduced problem 1	Reduced problem 2
Relevant set of scenarios	Number of scenarios	1	30	436
	Max. number of simultaneous disruptions	0	1	2
	Probability of scenarios	50.3%	85.4%	98.5%
Expected costs for scenario set	Investment (MM $)	18.47	18.78	21.56
	Total (MM $):	**34.09**	**48.68**	**53.85**
Solution	Optimality gap for scenario set	0%	0.44%	0.87%
	Full problem upper bound	**57.40**	**56.32**	**55.89**
	Full problem lower bound	**48.15**	**52.43**	**53.81**
Computational statistics	No. of constraints	11,849	304,256	4,397,984
	No. of continuous variables	10,080	251,186	3,626,670
	No. of binary variables	29	29	29
	Solution time	0.1 min	**289 min[a]**	**7,453 min[a]**

[a]: Strengthened Benders multi-cut

Economics vs. performance?

Multiobjective Optimization Approach

Bi-level optimization

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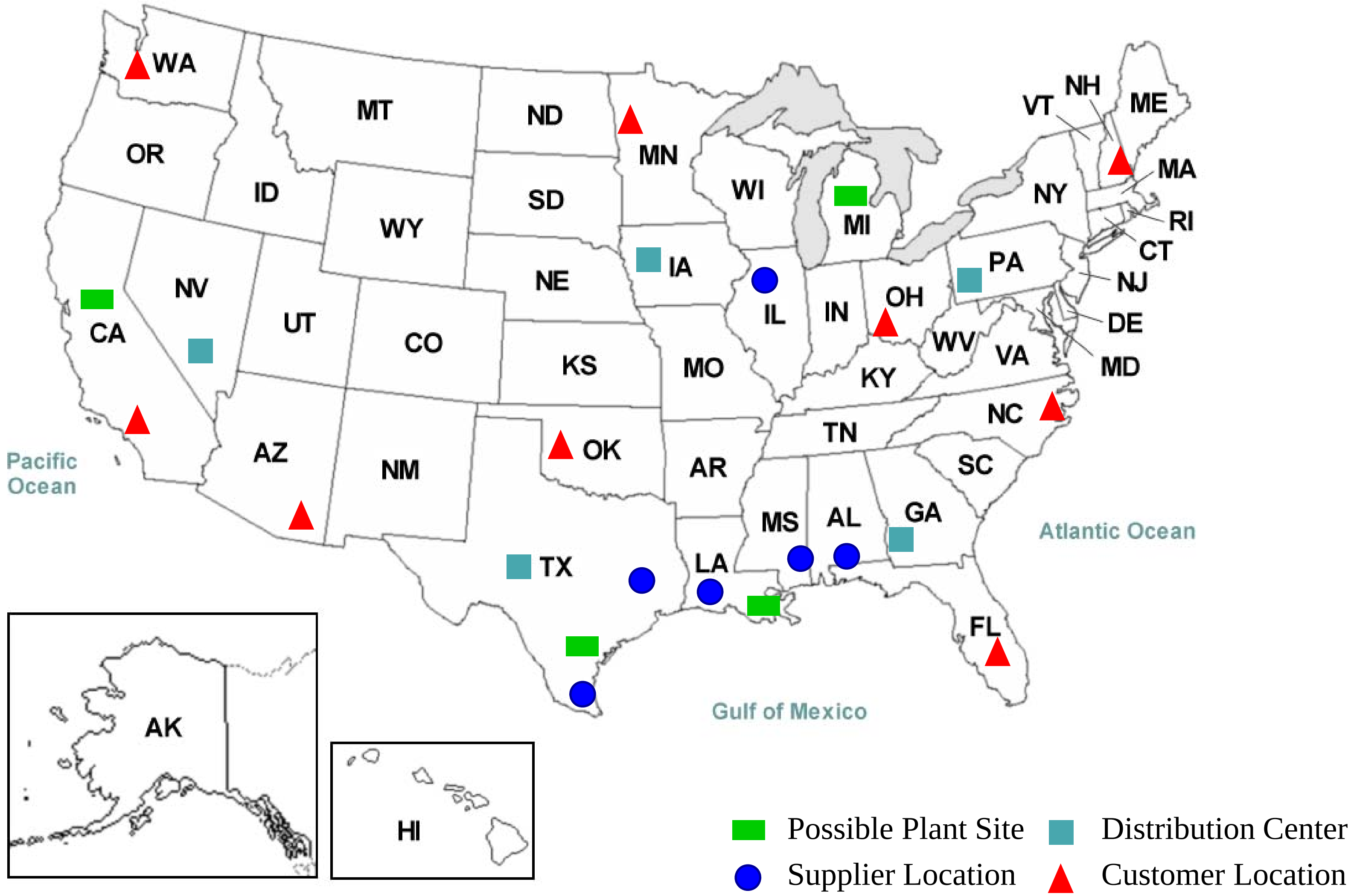
Optimal Design of Responsive Process Supply Chains

CAPD CENTER

Objective: design supply chain polystyrene resisns under responsive and economic criteria

You, Grossmann (2008)



Production Network of Polystyrene Resins

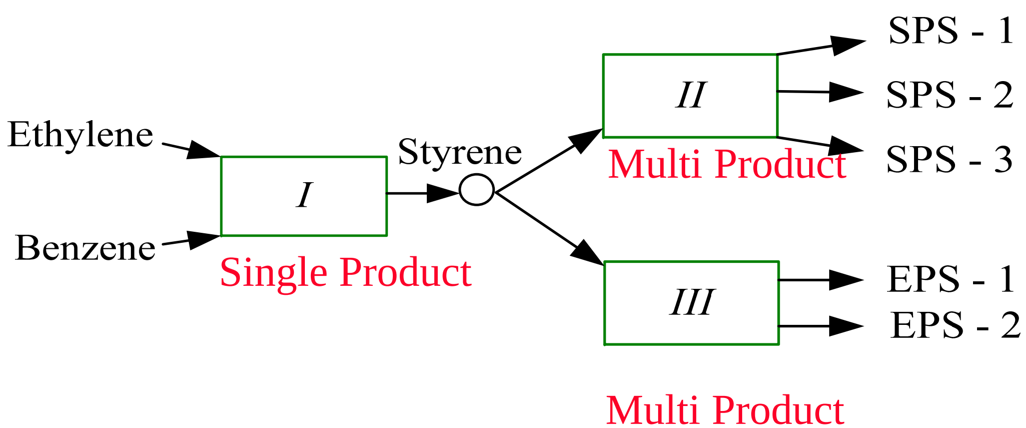
Three types of plants:

Plant *I*: *Ethylene + Benzene* ⟶ *Styrene* (1 products)

Plant *II*: *Styrene* ⟶ *Solid Polystyrene (SPS)* (3 products)

Plant *III*: *Styrene* ⟶ *Expandable Polystyrene (EPS)* (2 products)

Basic Production Network



Source: Data Courtesy Nova Chemical Inc. http://www.novachem.com/

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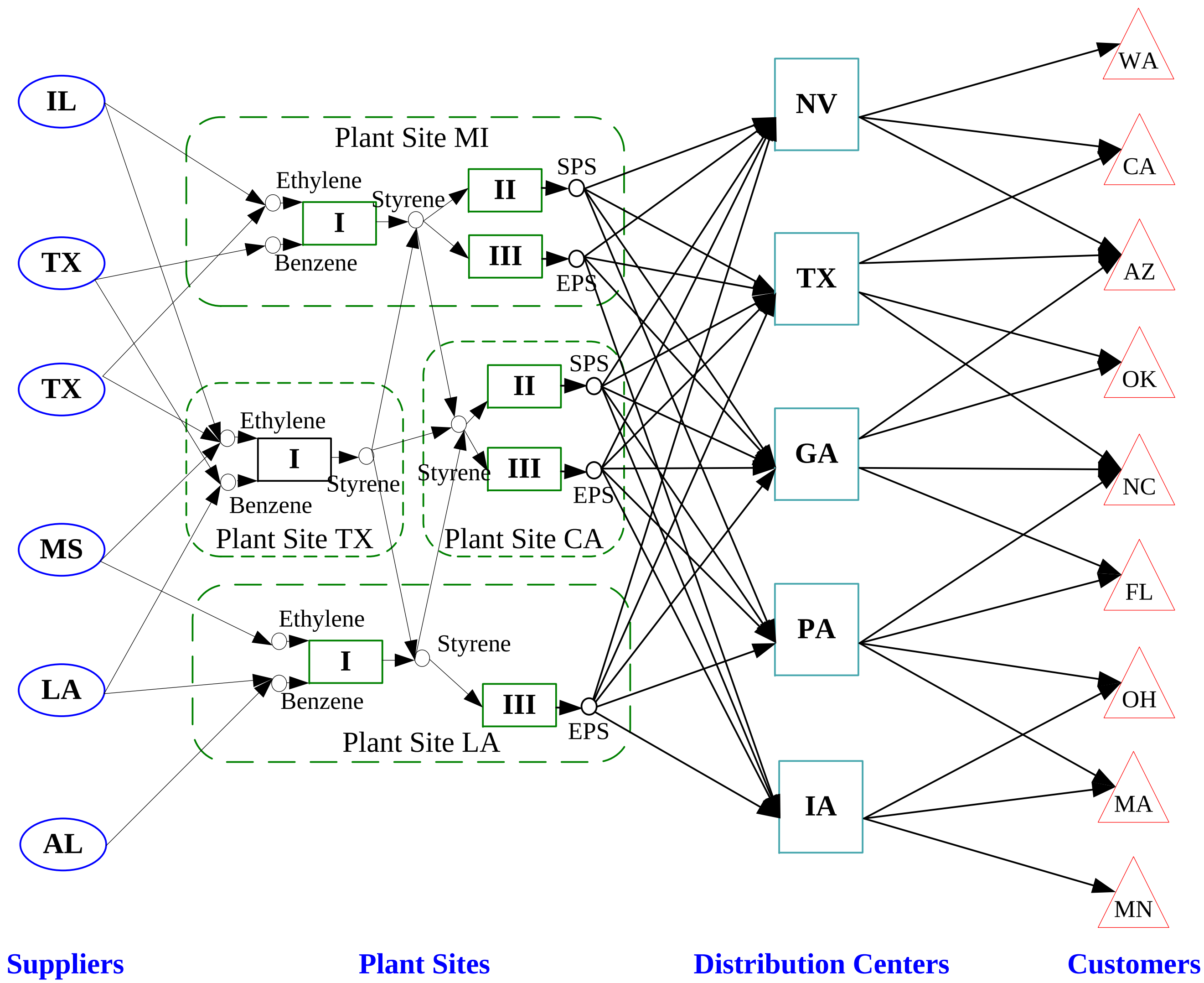
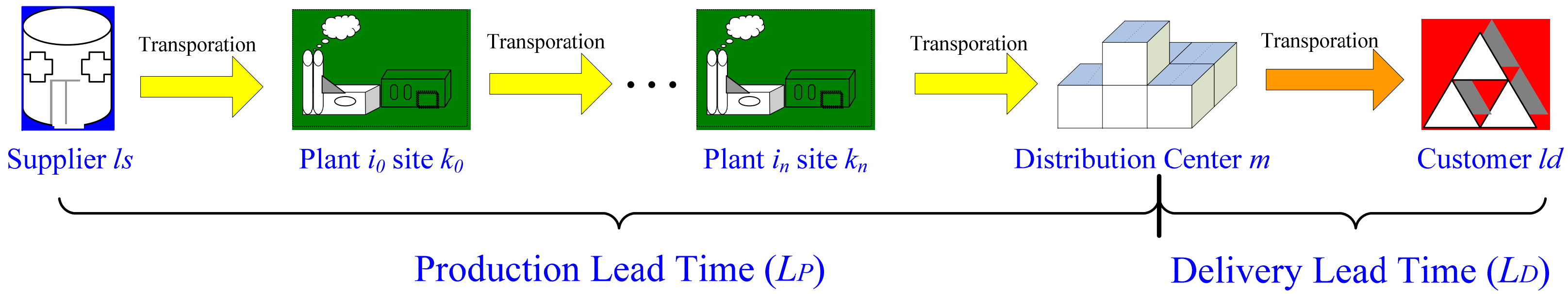
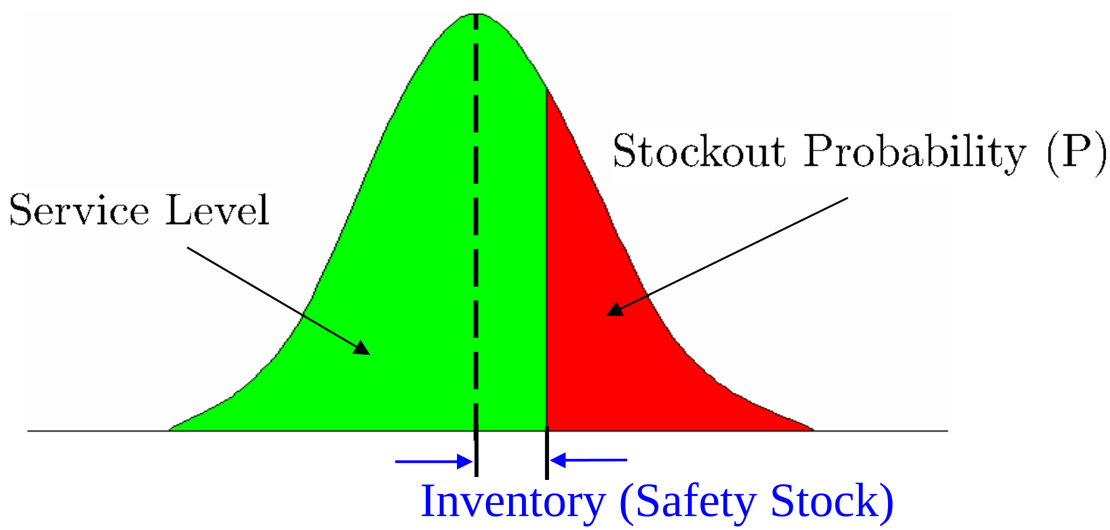
Potential Network Superstructure



Lead Time under Demand Uncertainty





$$\text{Expected Lead Time} = L_D + \text{P(Stockout)} \cdot L_P$$

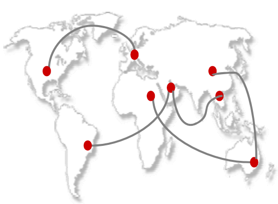
Bi-criterion Multiperiod MINLP Formulation

Choose Discrete (0-1), continuous variables

- Objective Function:
  - Max: Net Present Value
  - Min: Expected Lead time

  } Bi-criterion

- Constraints:
  - Network structure constraints



Suppliers – plant sites Relationship

Plant sites – Distribution Center

Input and output relationship of a plant

Distribution Center – Customers

Cost constraint

  - Operation planning constraints

Production constraint

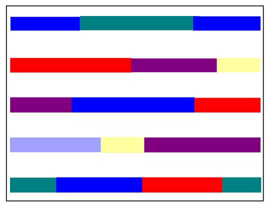
Capacity constraint

Mass balance constraint

Demand constraint

Upper bound constraint

  - Cyclic scheduling constraints



Assignment constraint

Sequence constraint

Demand constraint

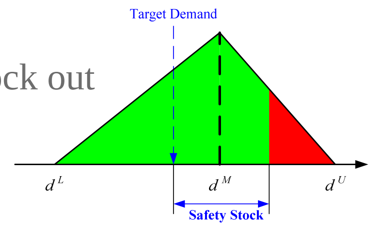
Production constraint

Cost constraint

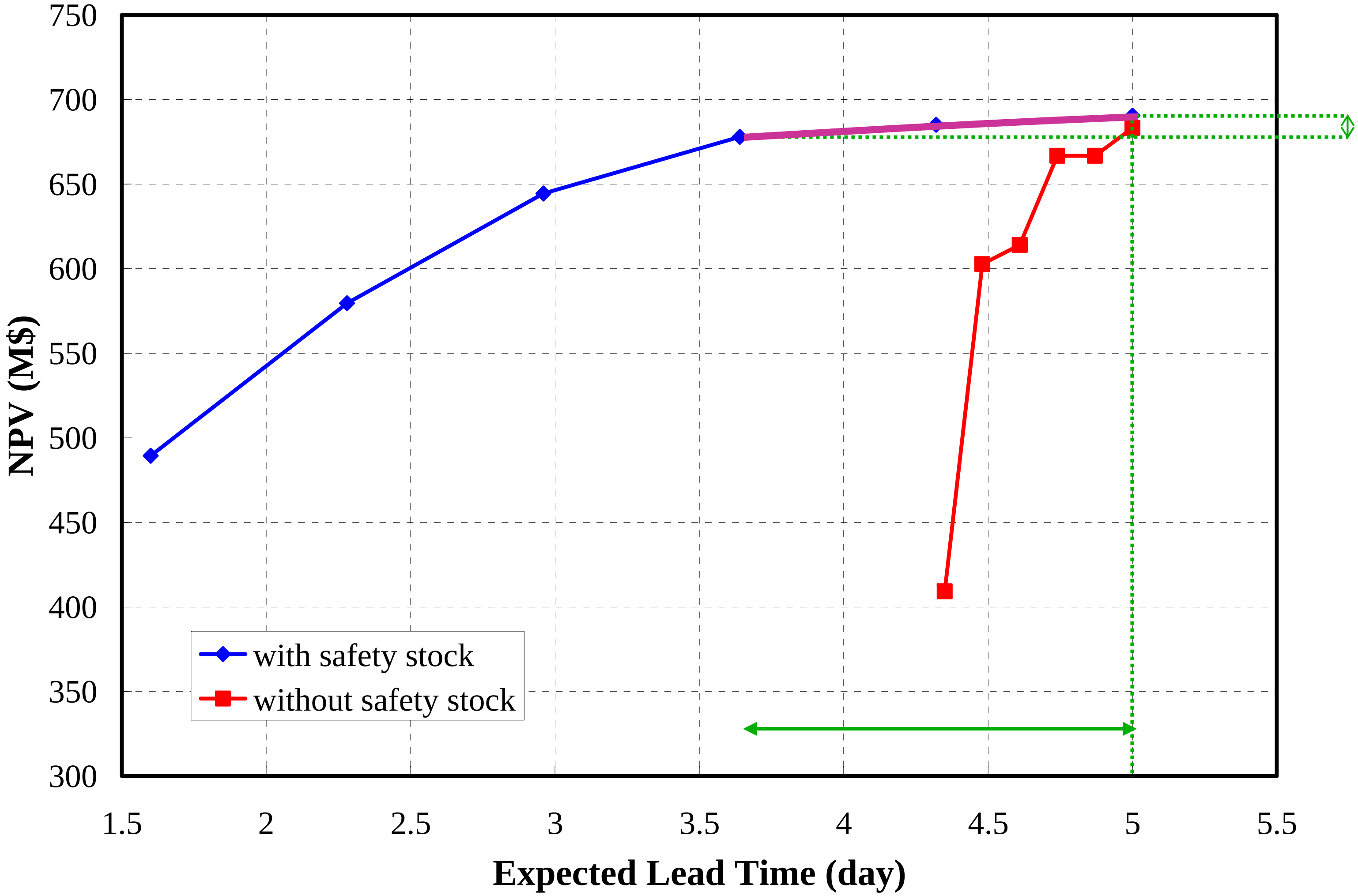
  - Probabilistic constraints

Chance constraint for stock out

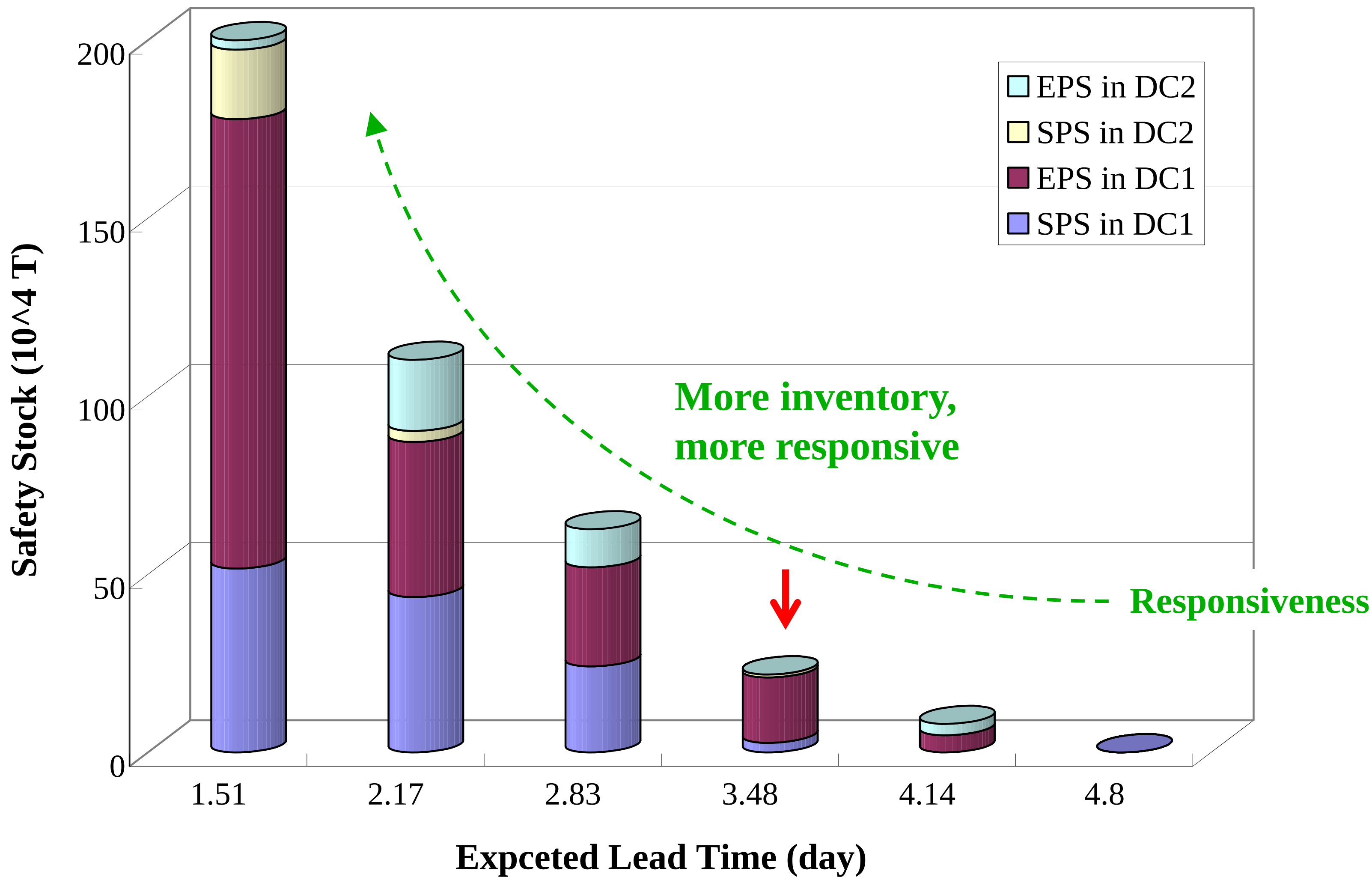
(reformulations)



Pareto Curves – with and without safety stock



Safety Stock Levels - Expected Lead Time



Bi-level Optimization for Capacity Planning in Industrial Gas Markets

Motivation

Industrial gas markets are dynamic:

- Suppliers must anticipate demand growth
- Most markets are served locally

Capacity expansion is a major strategic decision:

- Requires large investment cost
- Benefits are obtained over a long horizon

} **Optimization**

Benefits are sensitive to market behavior:

- Market preferences
- Economic environment

} **Variability**

Sensitivity can be reduced by assuming rational behavior:

- Producers try to maximize their profit
- Markets try to minimize their cost

} **Bilevel optimization**

Need to model the conflicting interests of producer and markets

Problem Statement

Given:

- Set of capacitated plants and candidate locations for new plants from leading supplier
- Set of plants from independent suppliers with limited capacity
- Rational markets that select their suppliers according to their own objective function
- Deterministic demands over the time horizon

Maximize net present value (NPV):

- Determine expansion plan
- Considering optimal distribution strategy in each time-period

Bilevel Approach (MILP)

Capacity expansion planning with rational market:

Plants are divided in two: plants from leading supplier (I^1) and plants from independent suppliers (I^2)

$$\max \ NPV = \sum_{t \in T} \frac{1}{(1+R)^t} \left\{ \sum_{i \in I^1} \sum_{j \in J} P_{t,i,j} y_{t,i,j} - \sum_{i \in I^1} \left[A_{t,i} v_{t,i} + B_{t,i} w_{t,i} + E_{t,i} x_{t,i} + \sum_{j \in J} \left(E_{t,i} y_{t,i,j} + G_{t,i,j,k} y_{t,i,j} \right) \right] \right\}$$

s.t.

$$w_{t,i} = V^o_{t,i} + \sum_{t'=1}^{t} v_{t,i} \qquad (\forall t \in T,\ i \in I^1) \qquad \text{Invest in new plants}$$

$$x_{t,i} \le w_{t,i} \qquad (\forall t \in T,\ i \in I^1) \qquad \text{Expand only open plants}$$

$$c_{t,i} = C^o_{t,i} + \sum_{t'=1}^{t} H x_{t-1,i} \qquad (\forall t \in T,\ i \in I^1) \qquad \text{Capacity expansion}$$

$$\min \sum_{t \in T} \frac{1}{(1+r)^t} \left[\sum_{j \in J} \sum_{i \in I} P_{t,i,j} y_{t,i,j} \right]$$

s.t.

$$\sum_{j \in J} y_{t,i,j} \le c_{t,i} \qquad (\forall t \in T,\ i \in I^1) \qquad \text{Markets minimize cost paid}$$

$$\sum_{j \in J} y_{t,i,j} \le C_{t,i} \qquad (\forall t \in T,\ i \in I^2) \qquad \text{Capacity of plants from leader}$$

$$\sum_{i \in I} y_{t,i,j} = D_{t,j} \qquad (\forall t \in T,\ j \in J) \qquad \text{Capacity of independent plants}$$

$$c_{j,k}, y_{s,j,i,k} \ \ge \ 0\ ;\ \ v_{t,i},\ w_{t,i},\ x_j \in \{0,1\} \qquad (\forall t \in T,\ i \in I,\ j \in J) \qquad \text{All markets are satisfied}$$

Illustrative Example

Problem structure:

- 3 existing plants which can be expanded
- 1 new candidate plant
- 3 existing plants which can not be expanded
- 15 markets with deterministic demand for 1 commodity
- 20 time-periods (quarters)

Formulations:

- Single-level (***SL***): leader selects the markets to satisfy
- Single-level evaluation (***SL-eval***): evaluation of single-level investment decisions in a market driven environment
- Bilevel KKT (***KKT***): KKT reformulation of the bilevel problem
- Bilevel Primal-Dual (***P-D***): Primal-dual reformulation of the bilevel problem

Results

Computational statistics:

Statistics	***SL***	***SL-eval***	***KKT***	***P-D***
No. of constraints:	680	520	8,460	4,380
No. of continuous variables:	2,240	2,240	6,060	4,220
No. of binary variables:	240	0	3,080	240
Solution time (CPLEX):	0.10 s	0.02 s	193 s	5.72 s
Optimality gap:	0.1%	0.1%	0.1%	0.1%

Results:

Items of objective function	***SL***	***SL-eval***	***KKT***	***P-D***
Income from sales [MM$]:	1,171	805	794	794
Investment in new plants [MM$]:	0	0	0	0
Capacity expansion cost [MM$]:	199	199	58	58
Maintenance cost[MM$]:	94	94	94	94
Production cost[MM$]:	424	292	279	279
Transportation cost[MM$]:	14	8	9	9
Total NPV [MM$]:	**440**	**212**	**354**	**354**
Market cost[MM$]:	**1,239**	**1,234**	**1,234**	**1,234**

Bilevel optimization yields **67% higher NPV** **(354 *vs* 212 million)** when compared to single-level expansion strategy

Conclusions

1. **Enterprise-wide Optimization (EWO) of great industrial interest**
   Great economic impact for effectively managing complex supply chains

2. **Mathematical Programming: major modeling framework**

3. **Key components in EWO: Planning and Scheduling**
   Modeling challenge:
   Multi-scale modeling (temporal and spatial integration)
   Linear vs. Nonlinear

4. **Computational challenges lie in:**
   a) Large-scale optimization models (decomposition, advanced computing)
   b) Handling uncertainty (stochastic programming)